Nonlinear Dynamic Analysis of Surface Mount Interconnects: Part I—Theory

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Solder joints are commonly used in surface mount technology microelectronics packaging. It is well known that the dominant failure mode for solder joints is thermal fatigue. When semiconductor devices are used in a vibrating environment, such as in automotive and military applications, dynamic stresses contribute to the failure mechanism and in certain circumstances they can become the dominant failure cause. In this paper a unified constitutive model for Pb40/Sn60 solder joints is developed and then implemented in a finite element dynamic analysis procedure. The purpose of the material model and the implementation is to study the contribution of vibration induced strains to the fatigue life of solder interconnects in low cycle and high cycle fatigue. The proposed material model, which is based on the disturbed state concept (DSC), is used for a dynamic analysis of a solder joint in the following paper, Part II, Basaran and Chandaroy (1998).

Introduction

Pb40/Sn60 solder joints are extensively used for connecting surface mount components to substrates in microelectronics packaging. As the miniaturization of microelectronics devices is continuing and VLSI circuits are getting bigger, the real estate on a package is becoming more scarce. As a result, the use of surface mounting components with solder joints is steadily increasing. Steinberg (1988) reports that ''solder is so important in this process that many people in the industry claim that the reliability of the electronic equipment amounts to the reliability of the solder joints''.

The solder joint attaches the components, such as a chip carrier, to a substrate while at the same time providing electrical and thermal continuity and mechanical support to the upper layer. It is well known that the dominant failure mode for solder joints is low cycle thermal fatigue. This is caused by the thermal cycling of the semiconductor device and the coefficient of thermal expansion mismatch between the soldered layers (Lau and Rice, 1985). The literature is scarce on the subject of the contribution of vibration damage to the overall fatigue life of solder joints. This is probably due to the fact that, according to popular opinion, dynamic loading is believed to cause elastic strains and only makes a secondary contribution to the fatigue damage (Blanks, 1976; Markstein, 1987; Steinberg, 1988; Suhir and Lee, 1988; Barker et al., 1990; Pitarresi and Akanda, 1993; Lau et al., 1995; Barker and Sidhart, 1995; Darbha et al., 1996). The U.S. Air Force reported that vibration and shock cause 20 percent of the mechanical failures in airborne electronics (Markstein, 1987). Barker et al. (1990) presents a generalized strain versus life approach model for combined vibrational and thermal fatigue of solder joints in which their results strongly suggest that strains caused by vibrations must be included in any reliability prediction model developed for solder joint reliability.

To study the dynamic behavior of solder joints, a damage mechanics based constitutive model is developed and implemented in a finite element procedure. Due to the viscous characteristics of the Pb40/Sn60 solder alloy, the model utilizes the theory of viscoplasticity.

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Constitutive Model

The constitutive model proposed for the Pb40/Sn60 is based on the disturbed state concept, (DSC). The DSC is a material modeling approach which treats the material as a two phase mixture and takes into account the effect of the damaged and the intact parts in the determination of the overall global response of the material.

The DSC allows for incorporation of the microstructural changes and the resulting micromechanisms into the constitutive tensor. Solder alloys are usually heterogenuous and often have flaws/discontinuities that form during the reflow process. The solder material cannot be treated as a continuum. Hence, it is necessary to introduce the discontinuous nature of the material into the constitutive model. When the solder joint is subjected to cycling shear strain due to thermal effects and/or vibrations, microstructural changes (dislocations) take place in the solder. Initially, the material is mostly in the intact state. As the external disturbances increase, the material transforms from the intact state to the damaged state. Then, at any given time, the material is composed of randomly distributed clusters of the material in the intact and damaged states. Consequently, the observed response of the material is defined by a combination of the response of the intact part and the response of the damaged part.

The response of the material in the intact part will be different than in the damaged part. In general the stresses will be higher in the intact part and the strains will be higher in the damaged part. Due to the stress differential between the damaged part and the intact part of the material a material moment exists, similar to the Cosserat and Cosserat (1909) continuum definition. Furthermore, due to having different strains in the intact part and in the damaged part there is a relative strain in the material. The relative motion within the material accounts for an additional energy dissipation mechanism. As a result, the DSC enables us to account for the energy of the material due to the intact part deformations, as in the traditional continuum mechanics. It also allows accounting for the energy dissipated due to the relative motion, as well as for the energy dissipated due to the material moment. The DSC therefore represents an accurate characterization of energy dissipation systems by allowing for the incorporation of microstructural mechanisms in the material.

The DSC is a particularly powerful material characterization approach for predicting solder alloy behavior because Pb/Sn is a two-phase alloy containing Pb-rich and Sn-rich phases, Bonda and Noyan (1996). Frear et al. (1995) report that the lead-rich phase regions in Pb/Sn solder alloys has been observed to correlate strongly with strain localization and subsequent failure. Lead-rich and tin-rich phases have different stress-strain responses (Frear et al., 1995). Therefore, having a two phase material definition in the DSC makes it more effective for characterizing the behavior of Pb/Sn solder alloys.

Formulation of the DSC

The incremental stress-strain equation is given by Basaran (1998) as follows:

$$d\sigma_{ij}^{a} = C_{ijkl}^{\mathrm{DSC}} d\epsilon_{kl}^{\mathrm{intact}}, \tag{1}$$

where $d\sigma_{ij}^{a}$ is the incremental average stress tensor, $d\epsilon_{kl}^{\mathrm{intact}}$ is the incremental strain tensor for the intact part, and the DSC tangential constitutive tensor is given by

$$C_{ijkl}^{\mathrm{DSC}} = [(1-D)C_{ijkl}^{\mathrm{intact}} + (1+\omega)DC_{ijkl}^{\mathrm{damaged}} + (\sigma_{ij}^{\mathrm{damaged}} - \sigma_{ij}^{\mathrm{intact}})R_{kl}], \tag{2}$$

where the damage evolution function D is given by

$$D = (1 - e^{-A\xi_D^Z}). \tag{3}$$

A and Z are material constants and $\xi_D = \int \sqrt{de_{ij}^{p} de_{ij}^{p}}$ is the trajectory of the deviatoric plastic strain.

$$R_{st} = \frac{[D_u A Z \xi_D^{Z-1} e^{-A\xi_D^Z}] \dfrac{\partial F}{\partial \sigma_{u\nu}} C^{e} u\nu st \left(\dfrac{\partial F}{\partial \sigma_{ij}} \dfrac{\partial F}{\partial \sigma_{ij}} - \dfrac{1}{3} \dfrac{\partial F}{\partial \sigma_{ij}} - \dfrac{\partial F}{\sigma_{ij}} \right)^{0.5}}{\left[\dfrac{\partial F}{\partial \sigma_{mn}} C^{e} mnpq \dfrac{\partial F}{\partial \sigma_{pq}} - \dfrac{\partial F}{\partial \xi} \left(\dfrac{\partial F}{\partial \sigma_{mn}} \dfrac{\partial F}{\partial \sigma_{mn}} \right)^{0.5} \right]} \tag{4}$$

$C_{ijkl}^{\mathrm{intact}}$ and $C_{ijkl}^{\mathrm{damaged}}$ are the tangential constitutive tensors for the intact and damaged parts, respectively; $\sigma_{ij}^{\mathrm{intact}}$ and $\sigma_{ij}^{\mathrm{damaged}}$ are the stress tensors for the intact and damaged parts, respectively; and ω is the empirical relative strain coefficient.

Equation (2) includes the effect of damage over the discontinuous material. If the damage, D, were not included, C_{ijkl}^{DSC}, would be reduced to the continuum mechanics formulation. Equation (2) includes the effect of the relative motion and the material moment within the material. The first term in Eq. (2) gives the Kachanov (1986) continuum damage mechanics formulation. The second term in the equation accounts for the relative strain within the material. This enables us to account for the energy dissipated within the material due to the different strains in the intact part and in the damaged part. The third term in Eq. (2) indicates different stresses in the two parts and includes a material moment effect due to the stress differential. The material moment in the DSC may be considered similar to the introduction of the moment in the Cosserat continuum, but it differs due to the fact that this moment is intrinsically included in the constitutive model. On the other hand in the Cosserat continuum the moment is introduced in the equilibrium equations.

In the DSC it is assumed that the initial intact material changes its microstructural configuration continuously through a process of self-organization (Bak and Tang, 1989; Aifantis, 1989), and it approaches the damaged state. As the effect of the intact material on the overall response decreases, the effect of the damaged part increases. The transformation from the intact state to the damaged state is defined through a damage function given by Eq. (3).

The yield surface, F, used in the proposed constitutive model is given by,

$$F = \frac{J_{2D}}{P_a^{2}} + \left[(\alpha(\theta) - m(\theta)) \left(\frac{J_1 + R(\theta)}{P_a} \right)^2 \right], \tag{5}$$

where J_1 is the first invariant of the total stress tensor, J_{2D} is the second invariant of the deviatoric stress tensor, Pa is the atmospheric pressure, $R(\theta)$ is the material bonding stress, $m(\theta)$ is the ultimate stress state material constant, defined by $m = (\sqrt{J_{2D}}/J_1)_{ult}$, and α is the hardening parameter given by $\alpha = a_1/\xi^{\eta_1}$, where a_1 and η_1 are material constants and $\xi = \int \sqrt{d\epsilon_{ij}^{p} d\epsilon_{ij}^{p}}$ is the trajectory of total plastic strain. The DSC formulation and yield function presented above are different than the earlier versions of this approach reported in Basaran et al. (1998). The new version has reduced the number of material parameters by three.

Elasto-Viscoplastic Model

For Pb/Sn solder alloys the homologous temperature, T_g, at which viscous characteristics of a material become near strongest, is relatively low, when compared to the room temperature (300°K). Therefore, using a thermoviscoplastic formulation is a must to account for time dependent inelastic deformations and strain-rate effects (Basaran, 1996).

In most solder joint reliability analyses the elastic component of the strain is usually ignored. It is usually assumed that the elastic strain has a trivial effect on the fatigue life and that the inelastic strain is the dominant factor for a damage analysis. But, in the case of dynamic loading due to a high frequency of vibrations, the damage accumulates very quickly because one thermal cycle could contain millions of vibration cycles. Therefore, as demonstrated in the following paper (Part II), Basaran and Chandaroy (1998), and also by Barker et al. (1990), ignoring vibration induced strains, elastic or inelastic, can lead to serious errors.

The strain increment for an elastoviscoplastic problem can be separated into three parts,

$$d\epsilon_{ij} = d\epsilon_{ij}^{\theta} + d\epsilon_{ij}^{e} + d\epsilon_{ij}^{vp}, \tag{6}$$

where $d\epsilon_{ij}^{\theta}$, $d\epsilon_{ij}^{e}$ and $d\epsilon_{ij}^{vp}$ are the incremental thermal, elastic and, viscoplastic strain tensors, respectively. The viscoplastic strain rate defined by Perzyna (1966) is used,

$$\dot{\epsilon}_{ij} = \Gamma < \Psi\left(\frac{F}{F_o}\right) > \frac{\partial F}{\partial \sigma_{ij}}, \tag{7}$$

where, Γ is a material fluidity parameter, F is the yield function given by Eq. (5), and F_o is a reference stress state, e.g., uniaxial yield stress, and σ_{ij} is the total stress tensor. The function $\Psi(F/F_o)$ is a positive monotonically increasing function determined from experimental data and is given by,

$$\left\langle \Psi\left(\frac{F}{F_o}\right) \right\rangle = \left\{ \begin{array}{ll} 0 & \text{if } F \le 0 \\ \left(\dfrac{F}{F_0}\right)^N & \text{if } F > 0 \end{array} \right\}, \tag{8}$$

where N is a material constant.

Finite Element Method Formulation

The constitutive model was implemented in a finite element procedure. For the displacement based finite element method, the equilibrium equation in incremental form is given by Bathe (1996),

$$\int_V [B]^T \{ d^{r}\sigma_{n+1}^{a} \} dV = \{ Q_{n+1} \} - \int_V [B]^T \{ ^{r-1}\sigma_n \} dV, \tag{9}$$

where $[B]$ is the strain-displacement transformation matrix, $\{d\sigma^{a}\}$ is the vector of average stress increments, V is volume,

n is the load step number, r is the iteration number, and $\{Q_{n+1}\}$ is the vector of nodal external loads. Implementing the DSC formulation in Eq. (9) yields,

$$\int_V [B]^T[{}^rL_n^{evp\theta}][B]dV\{d^rq_{n+1}\} = \{Q_{n+1}\} - \int_V [B]^T\{{}^{r-1}\sigma_n^a\}dV - \int_V [B]^T\{{}^{r-1}\sigma_n^c - {}^{r-1}\sigma_n^i\}dD_n dV + \int_V \Delta t_n[B]^T[{}^{r-1}L_n^{evp\theta}]\{{}^{r-1}\dot{\epsilon}_n^{vp\theta}\}dV + \int_V \Delta t\chi d\theta_n[B]^T[{}^rL_n^{evp\theta}][{}^{r-1}G_2]_n\{\bar{I}\}dV + \int_V \alpha_T d\theta_n[B]^T[{}^rL_n^{evp\theta}]\{\bar{I}\}dV, \quad (10a)$$

where $\{d^rq_n\}$ is the vector of nodal displacement increments, Δt is the time increment for viscoplasticity integration, χ is the time integration coefficient for viscoplasticity, $d\theta$ is the temperature increment, α_T is the coefficient of thermal expansion, $\{I\}$ is a unit vector, and $[G_2] = [\partial\dot{\epsilon}^{vp\theta}/\partial\theta]_n$, and

$$[{}^rL_n^{evp\theta}] = [(1 - D_n)[{}^i_rC_n^{evp\theta}] + D_n(1 + \omega)[{}^c_rC_n^{evp\theta}]]. \quad (10b)$$

In order to introduce the dynamic inertia and damping forces into the finite element equation, we can write the equation of motion for the material nonlinear case as follows:

$$[M]\{(\ddot{q}_{n+1} + (\ddot{q}_g)_{n+1})\} + [C]\{\dot{q}_{n+1}\} + [K]\{(q_n + dq_{n+1})\} = Q_{n+1}, \quad (11)$$

where q, $\dot{q}$, and $\ddot{q}$ are the nodal displacement, velocity, and acceleration vectors, respectively, and $\ddot{q}_g$ is the base acceleration vector; $[M]$ is the mass matrix, $[C]$ is the damping matrix, and $[K]$ is the stiffness, given by

$$K = \int_V [B]^T[{}^rL_n^{evp\theta}][B]dV. \quad (12)$$

Eq. (11) can be rewritten as follows:

$$[M]\{\ddot{q}_{n+1}\} + [C]\{\dot{q}_{n+1}\} + [K]\{dq_{n+1}\} = \{Q_{n+1}\} - \int_V [B]^T\{\sigma_n\}dV - [M]\{(\ddot{q}_g)_{n+1}\}. \quad (13)$$

Introducing the dynamic terms into Eq. (10) yields

$$[M]\{{}^r\ddot{q}_{n+1}\} + [{}^{r-1}C_n]\{{}^r\dot{q}_{n+1}\} + [{}^{r-1}K_n]\{d^rq_n\} = \{Q_{n+1}\} - \int_V [B]^T\{{}^{r-1}\sigma_n^a\}dV - \int_V [B]^T\{{}^{r-1}\sigma_n^c - {}^{r-1}\sigma_n^i\}dD_n dV + \int_V \Delta t_n[B]^T[{}^{r-1}L_n^{evp\theta}]\{{}^{r-1}\dot{\epsilon}_n^{vp\theta}\}dV + \int_V \Delta t\chi d\theta_n[B]^T[{}^rL_n^{evp\theta}][{}^{r-1}G_2]_n\{\bar{I}\}dV + \int_V \alpha_T d\theta_n[B]^T[{}^rL_n^{evp\theta}]\{\bar{I}\}dV - [M]\{\ddot{q}\}_{n+1}. \quad (14)$$

In the study, Rayleigh damping was used (Clough and Penzien, 1993).

$$[C] = \alpha[M] + \beta[K], \quad (15)$$

where α and β are Rayleigh damping coefficients and are determined from two unequal frequencies of vibration that have been obtained by doing an eigenvalue analysis in ANSYS.

Using Newmark's (implicit) scheme (with coefficients $\delta = \frac{1}{2}$ and $\alpha = \frac{1}{4}$), for a time domain integration of the equation of motion yields the following equation:

Table 1 Elastic and inelastic material properties for Pb40/Sn60

E (GPa)	ν	a_1	η_1	m	ω	R (GPa)	Γ	N	A	Z
15.2	0.4	2.9 E-6	0.62	8.0 E-4	0.1	288	1.8	2.67	0.1	0.68

$$\left[[M]\frac{4}{\Delta t_d^2} + [C_n]\frac{2}{\Delta t_d} + [{}^{r-1}K_n]\right]\{{}^rdq_{n+1}\} = = \{Q_{n+1}\} - \int_V [B]^T[{}^{r-1}\sigma_n^a]dV - -\int_V [B]^T\{{}^{r-1}\sigma_n^c - {}^{r-1}\sigma_n^i\}dD_n dV + \int_V \Delta t_n[B]^T[{}^{r-1}L_n^{evp\theta}]\{{}^{r-1}\dot{\epsilon}_n^{vp\theta}\}dV + \int_V \Delta t\chi d\theta_n[B]^T[{}^rL_n^{evp\theta}][{}^{r-1}G_2]_n\{\bar{I}\}dV + +\int_V \alpha_T d\theta_n[B]^T[{}^rL_n^{evp\theta}]\{\bar{I}\}dV - [M]\left[\frac{4}{\Delta t_d^2}(\{{}^{r-1}q_{n+1}\} - \{q_n\}) - \frac{4}{\Delta t_d}\{\dot{q}_n\} - \{\ddot{q}_n\}\right] - [C]\left[\frac{2}{\Delta t_d}(\{{}^{r-1}q_{n+1}\} - \{q_n\}) - \{\dot{q}_n\}\right] - [M]\{\ddot{q}_g\}_{n+1}. \quad (16)$$

It should be pointed out that there are two different time increments in Eq. (16), Δt and Δt_d. The first one is the time increment for the viscoplastic strain rate integration and the second one is for the time domain integration of the dynamic equation of motion.

Convergence

In a viscoplastic-time domain dynamic analysis there are two different convergence criteria that must be satisfied. One is the convergence of the viscoplastic equilibrium and the other one is the global convergence of the dynamic force equilibrium equation.

Convergence is checked with the following criteria for the dynamic equilibrium (Bathe, 1996):

$$\frac{\|\{{}^{r-1}F_{n+1}\} - [M]\{{}^{r-1}\ddot{q}_{n+1}\} - [C_n]\{\dot{q}_{n+1}\}\|}{\text{RNORM}} \le \text{RTOL} \quad (17)$$

and

$$\frac{\{{}^rdq_n\}(\{{}^{r-1}F_{n+1}\} - [M]\{{}^{r-1}\ddot{q}_{n+1}\} - [C_n]\{{}^{r-1}\dot{q}_{n+1}\})}{\{dq_n\}(\{F_n\} - [M]\{\ddot{q}_n\} - [C_n]\{\dot{q}_n\})} \le \text{ETOL}, \quad (18)$$

where $\{F\}$ is the right hand side of Eq. (10a), RTOL is a force tolerance and ETOL is an energy tolerance (both taken as 0.01). RNORM is the norm of the right hand side of the equilibrium equation (obtained from the first iteration of a load step).

The convergence to a steady state solution in viscoplastic problems can be monitored by checking the viscoplastic strain rate during each time step (Owen and Hinton, 1980),

$$\frac{\sum_{i=1}^{m} |\{d\epsilon_n^{vp}\}_i}{\sum_{i=1}^{m} |\{d\epsilon_1^{vp}\}_i|} \times 100 \le \text{Tolerance}, \quad (19)$$

where tolerance is taken as 1.

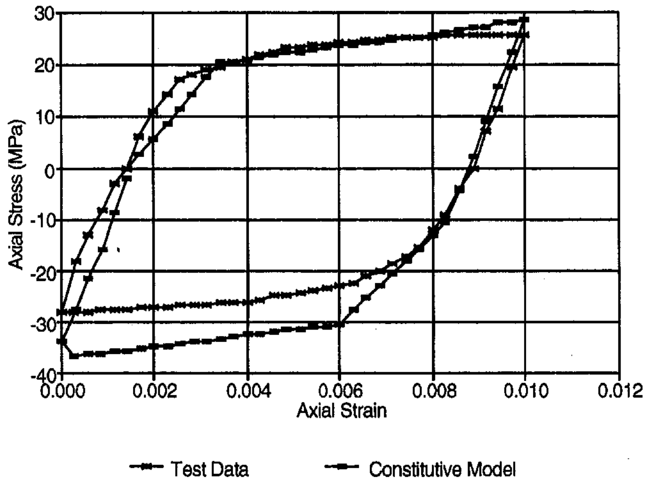

Fig. 1 Comparison of the test data versus the constitutive model backprediction

Verification of the Constitutive Model

In order to verify the constitutive model proposed in the preceding section, a laboratory test data reported in the literature was backpredicted. Guo et al. (1992) tested Pb36.37/Sn63.20 and 0.31 percent Sb by weight in an isothermal condition (25°C) under a total uniaxial extension strain-controlled test.

The dimension of the specimen tested by Guo et al. (1992) was 12 mm × 12 mm × 6.4 mm in the gauge section. The isothermal fatigue testing was performed in air under the uniaxial total strain control over the total strain range of 0.1 to 1 percent. The test was performed with a 120 s ramp time and the input strain versus time has a triangular form.

For the backprediction material constants were obtained from another test reported by Riemer (1990). Material properties are given in Table 1 are for Pb40/Sn60.

Figure 1 shows a comparison of the uniaxial extension test data versus the constitutive model prediction. The backprediction is very good, even though the material properties were obtained from a different experimental data and had a slightly different composition of Pb/Sn.

Conclusions

A simple unified damage mechanics based viscoplastic constitutive model has been proposed to characterize the behavior of the Pb/Sn solder alloy. The model was implemented in a finite element procedure for a time domain material nonlinear dynamic analysis of solder joints. Incorporating the damage formulation into the constitutive model eliminates the need to perform a stress analysis and a fatigue analysis separately.

The proposed material model was verified against laboratory test data that was not used to obtain the material parameters. The results show that the constitutive model can characterize the material behavior very well.

The following paper, Part II, presents an application of the model to a parametric study, a dynamic analysis of a surface mount technology solder joint.

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