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Thermomechanical Stress Analysis of Multi-Layered Electronic Packaging

An accurate estimate of thermal stresses in multilayered microelectronics structures along the bonded interfaces is crucial for design and prediction of delamination-related failures. Compared with a numerical method, analytical closed-form solution can offer a more rapid method to obtain the stresses at the interfaces. An analytical model for ply-level sub-laminate analysis is investigated in this paper. The theory presented treats each layer as a beam-type plate with orthotropic material properties. As an example, the results are shown for a three-layer beam problem with special orthotropic material properties. Analytical model results are compared with the finite element analysis results, as a first order approximation. [DOI: 10.1115/1.1535446]

Introduction

In electronic packaging, materials with different coefficient of thermal expansion (CTE) and mechanical properties are bonded together to form laminated structures, such as power electronics devices, circuit boards and semiconductor devices. Thermal stresses that occur due to CTE mismatch of the adhesively joined materials, during manufacturing, machining, and field use can result in delamination failure. An accurate estimate of thermal stresses in the interfaces plays an important role in the design and reliability studies of these devices.

A considerable amount of research is devoted to prediction of interfacial stresses in bonded dissimilar materials subjected to thermal loading. Even though numerical analysis procedures can be used for stress analysis of layered structures, the solution time in Central Processing Unit (CPU) is usually too long. In the microelectronics industry there is a need for a simple yet powerful analytical method to determine interfacial stresses in layered structures quickly and accurately.

Timoshenko [1] is the first one to study stresses in layered structures. He used elementary beam theory to obtain the curvature of a bimetallic beam due to a uniform temperature change. Grimado [2] considered the bonding layer as a third layer for investigating its effects on the two layers of a thermostat. In 1979, Chen and Nelson [3] used force equilibrium equations to predict thermal stresses in bonded joints, and Suhir [4–11] improved on Timoshenko's bi-thermostat beam theory with relatively simple calculations using longitudinal and transverse interfacial compliances, widely known as Suhir solution, which is the most commonly used benchmark analytical procedure in the electronic packaging literature. In 1991, Pao and Eisele [12] extended Suhir's bimetal thermostat model to multilayered thin stacks without imposing any additional assumptions on the interface.

Chen et al. [13] took a significantly different approach, which satisfied the boundary conditions at the free edges of a laminated beam. The analysis was based on two-dimensional elasticity theory and the variational theorem of complementary energy [14] under the assumption of linear distribution of longitudinal normal strain through the thickness of each layer. A similar approach was applied by Williams [15] and showed good agreement with the results of Chen et al. [13]. Bogy [16,17], Hein et al. [18], and Yin [19] discussed stress singularities at the interfaces near the free edges. Such stress singularities cannot be directly determined by the standard elastic finite element analysis alone. Asymptotic analysis is needed around the junction point to determine the stresses in the near-tip stress field [20–22]. Shih and Asaro [21,22] studied elastic-plastic analysis of cracks on bi-material interfaces, where they showed mesh dependence of finite element analysis and necessity of asymptotic analysis. In real life, such stress singularity $(1/r)$ cannot exist, physically. Once the stress level reaches the yield strength of the material, ductile materials yield and brittle material cracks and stress is redistributed to neighboring points. Basaran and Zhao [23] have shown that when damage mechanics based elastic-viscoplastic material models are used in finite element method stress singularity ceases to be a significant issue.

The ply-level analysis was first proposed by Pagano [24,25], who used two different theories for modeling the layers and sublaminates. The layers were examined on a local basis assuming that each layer was represented as a homogeneous anisotropic plate in equilibrium independent of the laminate. The sublaminates were studied on a global level using an assumed displacement model [26]. The refined engineering theories [27–29] for homogeneous plates and laminated plates provided another alternative for the layer and sublaminate models.

The present paper is an extension of the model proposed by Valisetty [29] with emphasis on the stress behavior along the interfaces between layers due to thermal loading. FEM is used to compare the analytical model results. Basaran and Zhao [23] have shown that using elastic FEM without an asymptotic analysis near the free-edge junction to validate analytical models is not the best approach, considering mesh sensitivity of FEM for laminated structures, yet it can be used as a first approximation for qualitative comparison. This paper is the first in a series of papers, which will follow this one where we will compare model simulations with actual test data from Moiré interferometry measurements.

In the following section formulation is presented which allow calculation of displacement and stresses in a laminated assembly under thermal gradient.

Analytical Modeling

Consider an N-layer laminated beam-type plate as shown in Fig. 1. A summary of the basic equations for generic ply is given as follows. The overall equations of equilibrium and the constitutive equations for the beam type plate theory will constitute a set of $8N$ equations in terms of a number of variables ($2N$ displacements, N rotations, $3N$ force resultants, and $2N$ moment resultants). This set is supplemented by an additional $2(N-1)$ equations, which are necessary for the simultaneous solution of $2(N-1)$ interlaminar stresses, if the displacement continuity is enforced at $(N-1)$ laminar interfaces. Among this set of equations

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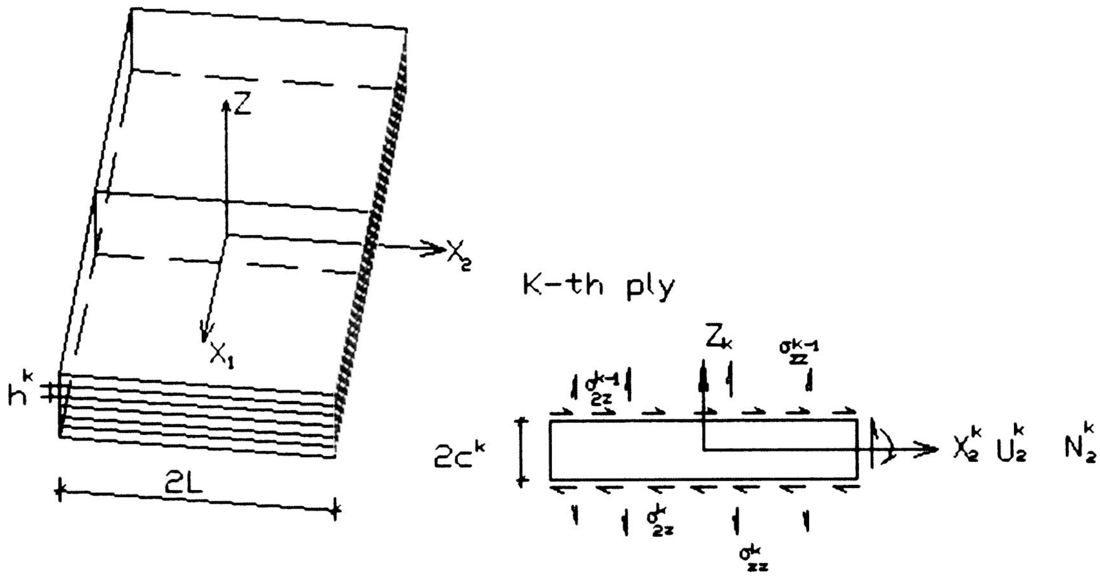


Fig. 1 Generic laminated beam-type plate

the force-resultant and moment-resultant variables can be eliminated with the aid of constitutive equations; these eliminations leave a set of $(5N-2)$ coupled differential equations to be solved for $2N$ displacement variables, N rotations, and $2(N-1)$ interlaminar stresses.

Starting from the classical plate theory and assuming that curvature in transverse direction is negligible, then overall equilibrium equations for a generic laminated beam-type plate in differential equation form for the k^{th} ply can given by

$$N_{2,2}^k + n_2^k = 0 \tag{1}$$

$$M_{2,2}^k + c_k m_2^k - Q_2^k = 0 \tag{2}$$

$$Q_{2,2}^k + q^k = 0 \tag{3}$$

where comma identifies differentiation w.r.t. the axis number after the comma. The difference in interfacial stresses between layers k and $k-1$ yield the stress imposed on each layer, which can be given by

$$n_2^k = \sigma_{2z}^{k-1}(x_2, c_k) - \sigma_{2z}^k(x_2, -c_k) \tag{4a}$$

$$m_2^k = \sigma_{2z}^{k-1}(x_2, c_k) + \sigma_{2z}^k(x_2, -c_k) \tag{4b}$$

$$q^k = \sigma_{zz}^{k-1}(x_2, c_k) - \sigma_{zz}^k(x_2, -c_k) \tag{4c}$$

where N_2, M_2, Q_2 are, respectively, the force, moment, and shear resultants per unit width of the plate, associated with the x_2-coordinate direction, and n_2, m_2, and q are the load terms for each layer due to the interfacial stress differentials between layers. $N_{2,2}$, $M_{2,2}$, $Q_{2,2}$ are, respectively, the derivatives of N_2, M_2, Q_2 with respect to x_2. The semi-thickness of the kth ply is c_k. Superscript k, which identifies the generic ply, will be dropped in the subsequent equations for convenience.

In microelectronics packaging the primary loading is thermal gradient and most layered structures have orthotropic material properties. Hence, we modify the Valisetty model to introduce the thermal loading and orthotropic material properties. In constitutive relations we also modify the coefficients $\overline{C}_{ij}$ and $\overline{C}_i$ to satisfy the orthotropic material properties we need for our analysis. As a result, we obtain the following constitutive relations:

$$\frac{N_i}{h} = -\overline{C}_{ij}\Delta T \alpha_j + \overline{C}_{i2} U_{2,2} + K_{ni} c n_{2,2} + K_{pi} p; \quad i,j = 1,2$$

$$\frac{M_i}{\overline{I}} = -\overline{C}_{i2} W_{,22} + K_{mi} m_{2,2} + K_{qi} q/c \tag{5a}$$

$$\Phi_2 + W_{,2} = \frac{c^2}{2\overline{I}} S_{44}\left(Q_2 - \frac{1}{3} c m_2\right) \tag{5b}$$

where

$$K_{mi} = (3\overline{C}_{i2} S_{3j} \overline{C}_{j2}/\overline{C}_{22} - 2\overline{C}_{i2} S_{44} + 2\overline{C}_i)/20; \quad i,j = 1 \text{ and } 2$$

$$K_{qi} = (3\overline{C}_{i2} S_{3j} \overline{C}_{j2}/\overline{C}_{22} - 12\overline{C}_{i2} S_{44} + 12\overline{C}_i)/20$$

$$K_{ni} = (\overline{C}_{i2} S_{3j} \overline{C}_{j2}/\overline{C}_{22} + \overline{C}_{i2} S_{44} + 2\overline{C}_i)/12$$

$$K_{pi} = \overline{C}_i/2$$

$$p = \sigma_{zz}(x_2, c) + \sigma_{zz}(x_2, -c)$$

$$h = 2c, \quad \overline{I} = 2c^3/3$$

$$\overline{C}_{11} = C_{11} - \frac{C_{13}C_{31}}{C_{33}}, \quad \overline{C}_{12} = C_{12} - \frac{C_{13}C_{32}}{C_{33}},$$

$$\overline{C}_{21} = C_{21} - \frac{C_{23}C_{31}}{C_{33}}, \quad \overline{C}_{22} = C_{22} - \frac{C_{23}C_{32}}{C_{33}} \tag{6}$$

$$\overline{C}_1 = \frac{C_{13}}{C_{33}}, \quad \overline{C}_2 = \frac{C_{23}}{C_{33}}$$

where C_{ij}: stiffness coefficients of orthotropic materials; α_j: coefficient of thermal expansion of kth layer in direction j; U, W: displacement components in the x_2 and z directions, respectively, at $z=0$ surface; and Φ_2: the rotation of a normal to the reference surface $(z=0)$.

Solution of the differential equations for the classical plate theory with beamlike behavior assumptions yield the following stress distribution equations:

$$\sigma_i = \frac{1}{h}N_i + \frac{1}{2h}K_i n_{2,2}(z^2 - c^2/3) + \frac{z}{\overline{I}}M_i$$

$$+ \frac{1}{6\overline{I}}K_i(cm_{2,2} + q)(z^3 - 3c^2z/5); \quad i,j = 1,2 \tag{7a}$$

$$\sigma_{2z} = \frac{z}{h}n_2 + \frac{c}{6\overline{I}}m_2(3z^2 - c^2) - \frac{1}{2\overline{I}}Q_2(z^2 - c^2) \tag{7b}$$

$$\sigma_{zz} = \frac{1}{2}p - \frac{1}{2h}n_{2,2}(z^2 - c^2) + \frac{z}{h}q - \frac{1}{6\overline{I}}(cm_{2,2} + q)(z^3 - c^2z) \tag{7c}$$

where

$$K_i = \overline{C}_{i2}S_{3j}\overline{C}_{2j}/\overline{C}_{22} + \overline{C}_{i2}S_{44} - \overline{C}_i; \quad i,j = 1,2 \tag{8}$$

σ_1: normal stress in the x_1 direction in any layer; σ_2: normal stress in the x_2 direction, in any layer; σ_{2z}: shear stress in the x_2-z-plane; and σ_{zz}: transverse normal stress in the thickness coordinate z direction, a.k.a. peeling stress.

Introduction of thermal strain terms into Valisetty [29] model yields following equations for displacements for isothermal loading:

$$w = W + S_{3j}\left(N_j\frac{z}{h} + M_j\frac{z^2}{2\overline{I}}\right) + S_{3j}K_j\frac{1}{6h}n_{2,2}(z^3 - c^2z) + S_{3j}K_j$$

$$\times \frac{cm_{2,2} + q}{6\overline{I}}(z^4/4 - 3c^2z^2/10) + S_{33}\left(\frac{z}{2}p + \frac{z^2}{2h}q\right)$$

$$- S_{33}\frac{1}{6h}n_{2,2}(z^3 - 3c^2z) - S_{33}\frac{cm_{2,2} + q}{6\overline{I}}(z^4/4 - c^2z^2/2)$$

$$+ z\Delta T\alpha_z \quad j = 1,2 \tag{9a}$$

$$u_2 = U_2 - zW_{,2} - S_{3j}\left(N_j\frac{z^2}{2h} + M_j\frac{z^3}{6\overline{I}}\right)_{,2} - S_{3j}K_j$$

$$\times \left\{\frac{n_{2,22}}{6h}(z^4/4 - c^2z^2/2) + \frac{cm_{2,22} + q_{,2}}{6\overline{I}}(z^5/20 - c^2z^3/10)\right\}$$

$$- S_{33}\left\{\frac{z^2}{4}p_{,2} - \frac{1}{6h}n_{2,22}(z^4/4 - 3c^2z^2/2) + \frac{z^3}{6h}q_{,2}\right.$$

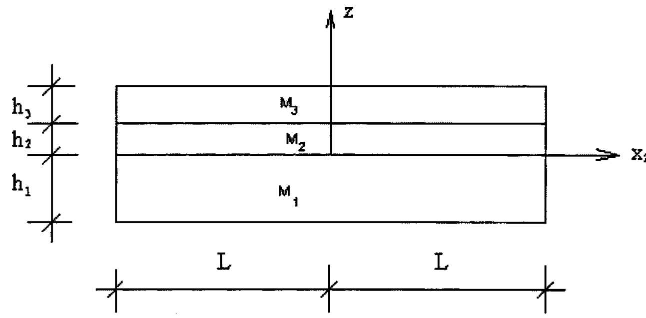
$$\left. - \frac{cm_{2,22} + q_{,2}}{6\overline{I}}(z^5/20 - c^2z^3/6)\right\}$$

$$+ S_{44}\left\{\frac{z^2}{2h}n_2 + \frac{1}{6\overline{I}}cm_2(z^3 - c^2z) - \frac{1}{6\overline{I}}Q_2(z^3 - 3c^2z)\right\} \tag{9b}$$

where w, u_2: the displacement components in the z and x_2 coordinate directions, respectively.

A Case Study

Laminated structures are commonly used in electronic packaging [4–12]. For the sake of simplicity in this study we will consider a three-layer laminated beam-type plate. The beam is subjected only to uniform temperature change ($\Delta T = 100°C$) causing stresses due to different thermal expansion coefficients. The geometry of the structure is shown in Fig. 2; orthotropic material properties and dimensions for the three-layered structure are given as well.



	M_1	M_2	M_3
E_1 (Gpa)	11	15	13
E_2 (Gpa)	140	138	16
E_3 (Gpa)	11	15	13
G_{12} (Gpa)	5.5	5.9	2.7
G_{13} (Gpa)	5.5	5.9	2.7
G_{23} (Gpa)	5.5	5.9	2.7
ν_{12}	0.29	0.21	0.16
ν_{13}	0.29	0.21	0.16
ν_{23}	0.3	0.21	0.16
α_1(e-6/°C)	0.36	0.9	0.5
α_2(e-6/°C)	28.8	23	18
α_3(e-6/°C)	28.8	23	18
h(mm)	0.508	0.0508	0.14
2L(mm)	15.24		

Fig. 2 Geometry and material properties of the analyzed structure

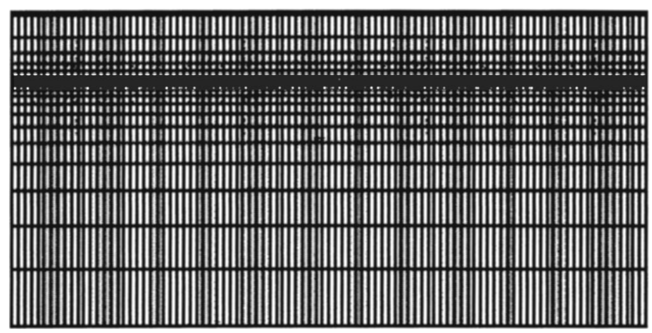


Fig. 3 FEM mesh design

Equations (2) and (3) may be combined to produce an equilibrium differential equation

$$M^k_{2,22} + cm^k_{2,2} + q^k_2 = 0 \tag{10}$$

where

M_2 = moment resultant associated with x_2 direction
c = ply semi-thickness
m_2 = $\sigma_{2z}(x_2, c) + \sigma_{2z}(x_2, -c)$
q_2 = $\sigma_{zz}(x_2, c) - \sigma_{zz}(x_2, -c)$

Assuming perfect displacement compatibility at the interfaces, the continuity requirement of displacements yields the following equations:

$$u^1_2\left(x_2, \frac{h_1}{2}\right) = u^2_2\left(x_2, -\frac{h_2}{2}\right) \tag{11a}$$

$$u^2_2\left(x_2, \frac{h_2}{2}\right) = u^3_2\left(x_2, -\frac{h_3}{2}\right) \tag{11b}$$

$$w^1\left(x_2, \frac{h_1}{2}\right) = w^2\left(x_2, -\frac{h_2}{2}\right) \tag{11c}$$

$$w^2\left(x_2, \frac{h_2}{2}\right) = w^3\left(x_2, -\frac{h_3}{2}\right) \tag{11d}$$

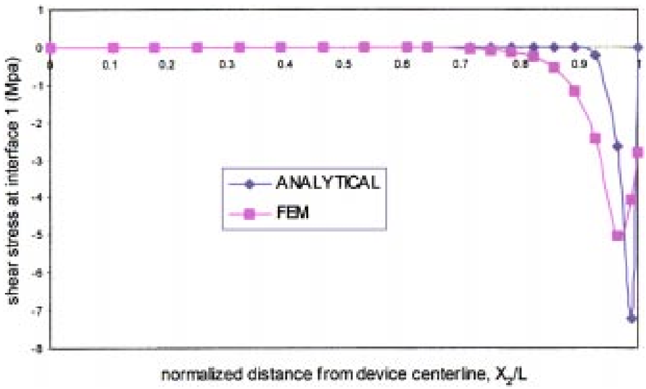


Fig. 4 Comparison of peeling stress at interface 1 between FEM and analytical solution

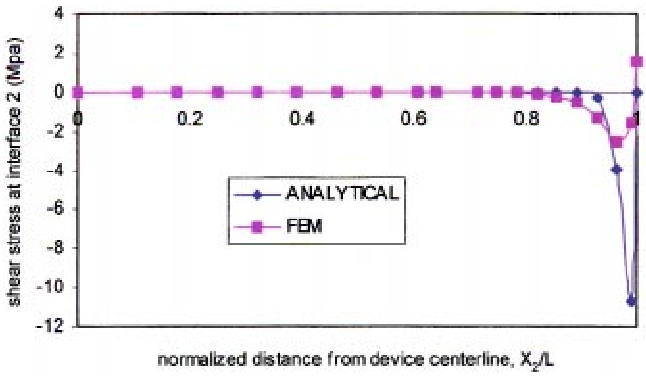


Fig. 5 Comparison of shear stress at interface 2 between FEM and analytical solution

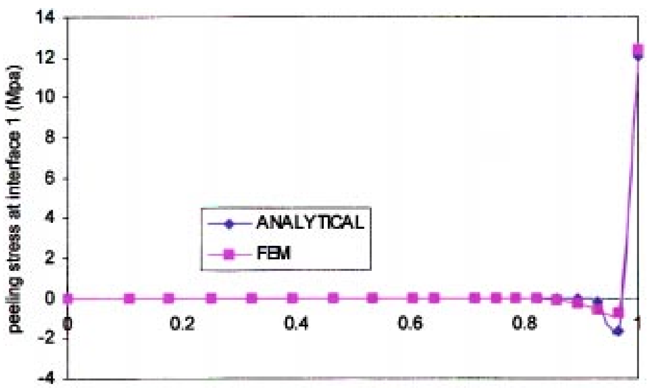


Fig. 6 Comparison of peeling stress at interface 1 between FEM and analytical solution

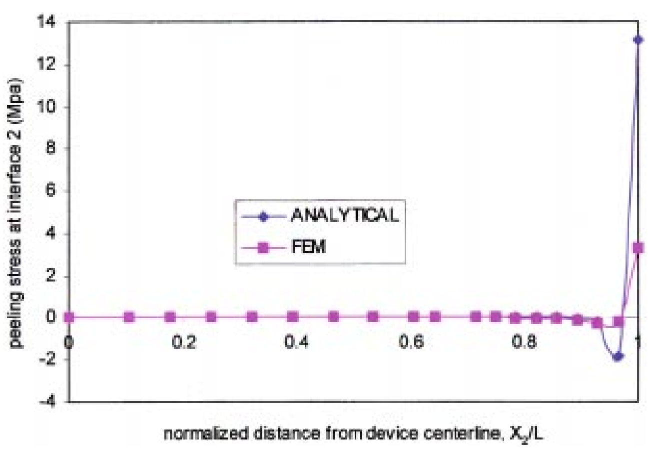


Fig. 7 Comparison of peeling stress at interface 2 between FEM and analytical solution

In the next version of this model, we plan to introduce interfacial compliance to relax the strict requirements given by Eqs. (11*a*,*b*,*c*,*d*).

There are 10 unknowns and 10 equations in Eqs. (10) and (11). These are W^1, W^2, W^3, U_2^1, U_2^2, U_2^3, σ_{zz}^1, σ_{zz}^2, σ_{2z}^1, and σ_{2z}^2

where
σ_{zz}^1 = peeling stress at interface 1
σ_{zz}^2 = peeling stress at interface 2
σ_{2z}^1 = shear stress at interface 1
σ_{2z}^2 = shear stress at interface 2

In order to solve the differential equation given by Eq. (10), we need to introduce boundary conditions. For this problem we will assume that boundary conditions can be given by: At $x_2=0$

$$\begin{aligned} &U_2^1=0, \quad \Phi_2^1=0, \quad Q_2^1=0 \\ &U_2^2=0, \quad \Phi_2^2=0, \quad Q_2^2=0 \\ &U_2^3=0, \quad \Phi_2^3=0, \quad Q_2^3=0 \\ &\sigma_{2z}(0,0)=0, \quad \sigma_{2z}(0,h_2)=0 \end{aligned} \tag{12a}$$

At $x_2=L$

$$\begin{aligned} &N_2^1=0, \quad M_2^1=0, \quad Q_2^1=0 \\ &N_2^2=0, \quad M_2^2=0, \quad Q_2^2=0 \\ &N_2^3=0, \quad M_2^3=0, \quad Q_2^3=0 \\ &\sigma_{2z}(L,0)=0, \quad \sigma_{2z}(L,h_2)=0 \end{aligned} \tag{12b}$$

Using hyperbolic method we can obtain the analytical solutions for the differential equation given by Eq. (10), ($\xi=x_2/L$)

$$\begin{aligned} \sigma_{2z}^1 = {} & 8.79097\times10^{-47}\ \mathrm{Sinh}[110.985\xi]\mathrm{Cos}[12.4421\xi] \\ & -2.87743\times10^{-33}\ \mathrm{Sinh}[79.7974\xi] \\ & -3.64717\times10^{-144}\ \mathrm{Sinh}[(332.378)\xi] \end{aligned}$$

$$\begin{aligned} \sigma_{2z}^2 = {} & 1.30455\times10^{-46}\ \mathrm{Sinh}[110.985\xi]\mathrm{Cos}[12.4421\xi] \\ & -4.27001\times10^{-33}\ \mathrm{Sinh}[79.7974\xi] \\ & -5.41227\times10^{-144}\ \mathrm{Sinh}[332.378\xi] \end{aligned}$$

$$\begin{aligned} \sigma_{zz}^1 = {} & 9.5428\times10^{-47}\ \mathrm{Cosh}[110.985\xi]\mathrm{Cos}[12.4421\xi] \\ & -2.24578\times10^{-33}\ \mathrm{Cosh}[79.7974\xi] \\ & -1.18567\times10^{-143}\ \mathrm{Cosh}[332.378\xi] \\ & -1.06981\times10^{-47}\ \mathrm{Sinh}[110.985\xi]\mathrm{Sin}[12.4421\xi] \end{aligned}$$

$$\begin{aligned} \sigma_{zz}^2 = {} & 1.04462\times10^{-46}\ \mathrm{Cosh}[110.985\xi]\mathrm{Cosh}[12.4421\xi] \\ & -2.4584\times10^{-33}\ \mathrm{Cosh}[79.7974\xi] \\ & -1.29792\times10^{-143}\ \mathrm{Cosh}[332.378\xi] \\ & -1.17109\times10^{-47}\ \mathrm{Sinh}[110.985\xi]\mathrm{Sin}[12.4421\xi] \end{aligned}$$

Results and Discussion

The example problem shown in Fig. 2 was also analyzed by ANSYS general-purpose finite element code. Figure 3 shows a mesh used for this study. The total number of elements is 8400. This is a fine mesh and would yield reasonable results according to an earlier study by Basaran and Zhao [23]. Figures 4 and 5 show the distribution of shear stress at the interfaces one and two, respectively, for both FEM and analytical solutions. Numeration of layers starts from bottom. Figures 6 and 7 show the distribution of peeling stress at the interfaces one and two, respectively, for both FEM and analytical solutions.

In Figs. 4–7 the term ''analytical solution'' refers to the solution we obtained by using the method suggested in this paper Eq. (13), which is essentially based on solving classical plate theory equilibrium differential equations for the given boundary conditions and curvature assumptions.

The distribution of the interfacial shear stress and peeling stress in interface 1 by the presented theory and FEM are in good agreement both qualitatively and quantitatively. On the other hand shear stresses and peeling stresses obtained by the analytical procedure and FEM differ significantly quantitatively yet they are similar qualitatively for interface 2. Basaran and Zhao [23] have shown that even though FEM is a crude procedure to check quantitative validity of an analytical model for layered structures. Yet it can be used to validate analytical procedures qualitatively. Linear-elastic FEM suffers from mesh sensitivity in layered structures where bi-material interfaces are present, Hadjesfandiari and Dargush [30], Basaran and Zhao [23]. Mesh sensitivity in layered elastic structures is mainly due to stress singularity near the free edge. Presently, work is underway to measure the strain field in the bi-material interfaces using high sensitivity Moiré interferometry with phase shifting. Comparing experimental results with the analytical results presented in this paper will be the subject of another paper. As a result of our Moiré measurements we will probably modify the analytical model proposed here.

Figures 4 to 7 indicate that interface stresses are zero or near zero from $X_2/L=0$ to $X_2/L=0.8$ with prescribed material properties. This is a numerical result, does not support earlier interface shear strain measurements conducted in solder joint interfaces by Zhao et al. [31,32]. Based on our solder joint experience, it is expected that strain in this region in our Moire interferometry measurements will be very small probably less than 10% of the maximum which is near the edge. The reason for zero strain is that the model and FEM assume a rigid bond, perfect displacement compatibility, between the layers; but in real life everything has stiffness, nothing is perfectly rigid. Therefore, the analytical model presented in this paper would not be applicable if the interface has a small stiffness. As the actual interface behavior deviates from Eq. (11) requirements, which many interfaces in electronic packaging do, this model would lose its accuracy.

Conclusions

An analytical procedure based on classical plate theory formulation for calculating interfacial shear and peeling stresses in layered structures under isothermal loading is presented. The method can take into account special orthotropic material properties. Comparison of analytical procedure results with finite element analysis results are in reasonably good agreement qualitatively. Compared to six hours of CPU time required in FEA, using the analytical procedure presented here results can be obtained in few minutes.

Acknowledgments

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