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Experimental damage mechanics of microelectronic solder joints under fatigue loading

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Abstract

Fatigue damage is a progressive process of material degradation. The objective of this study is to experimentally qualify the damage mechanism in solder joints in electronic packaging under thermal fatigue loading. Another objective of this paper is to show that damage mechanism under thermal cycling and mechanical cycling are very different. Elastic modulus degradation under thermal cycling, which is considered as a physically detectable quantity of material degradation, was measured by Nano-indenter. It was compared with tendency of inelastic strain accumulation of solder joints in Ball Grid Array package under thermal cycling, which was measured by Moiré interferometry. Fatigue damage evolution in solder joints with traditional load-drop criterion was also investigated by shear strain hysteresis loops from strain-controlled cyclic shear testing of thin layer solder joints. Load-drop behavior was compared with elastic modulus degradation of solder joints under thermal cycling. Following conventional Coffin–Manson approach, *S–N* curve was obtained from isothermal fatigue testing with load-drop criterion. Coffin–Manson curves obtained from strain-controlled mechanical tests were used to predict fatigue life of solder joints. In this paper it is shown that this approach underestimates the fatigue life by an order of magnitude. Results obtained in this project indicate that thermal fatigue and isothermal mechanical fatigue are completely different damage mechanism for microstructurally evolving materials.

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1. Introduction

The reliability concerns for solder joints are increasing exponentially with the increasing use of surface mount technology in microelectronics industry. Solder alloys are most commonly used bonding materials in electronic packaging, which provide electrical and thermal interconnection, as well as mechanical support. The temperature fluctuations due to device internal heat dissipation and ambient temperature changes, along with the coefficient of thermal expansion (CTE) mismatch between the soldered layers result in thermo-mechanical fatigue of the solder joints. Progressive damage in solder balls eventually leads to device failure. Fatigue life prediction of solder joints is critical to the reliability assessment of electronic packaging.

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The state-of-the-art method for thermal fatigue life prediction is based on using empirical relations, such as Coffin–Manson approach. Usually, the plastic strain range of a solder joint under thermal cycling is determined by finite element method. Then Coffin–Manson curves, which are obtained from isothermal mechanical testing, are used to predict the fatigue life of solder joints (Darveaux and Banerji, 1992). Usually this approach yields very conservative results for thermal fatigue life prediction of Ball Grid Array (BGA) packaging. Solder alloys as cast are thermodynamically unstable materials and microstructurally evolve into a stable equaixed configuration over time under strain and heat. As a result plastic strain accumulation in solder joints under thermal cycling is a nonlinear process, and plastic strain range of just one or several cycles cannot appropriately reflect the physical mechanism of fatigue damage evolution. The purpose of this paper is to prove that this state-of-the-art practice underestimates fatigue life significantly. Moreover, damage mechanism under mechanical loading is quite different than under thermal loading. In this paper we quantitatively show that *S–N* curves from isothermal loading cannot be used to predict thermal fatigue life.

Recently, numerous damage mechanics based models have been developed for the evaluation of thermomechanical fatigue reliability, which consider damage as an intrinsic material state (Ju, 1989; Lemaitre and Chaboche, 1990; Dasgupta et al., 1992; Hayakawa and Murakami, 1997; Basaran and Yan, 1998; Basaran and Tang, 2002; Basaran and Chandaroy, 1998; Chow and Yang, 1998; Qian et al., 1999). Most models use a set of internal state variables, known as damage variables, to describe the state of damage. With the introduction of damage variables at Gauss integration points in a finite element method, damage distribution all over the structure can be characterized adequately as a function of time. On the other hand, traditional approaches just give a number of cycles to failure prediction, which cannot reflect the progressive process of fatigue damage evolution due to growth and coalescence of microcracks. Furthermore, traditional fatigue theory of Coffin–Manson, which is not an inherent intrinsic approach, cannot give the damage distribution of structure under fatigue loading.

Herein damage is defined as the gradual degradation of material strength due to growth and coalescence of smeared microvoids or microcracks to initiate a single crack in the representative volume element under continuous load application. As an intrinsic material property, damage variable can be readily determined experimentally at the microscale, such as dislocation density or microcrack density. Nevertheless, presently it is not feasible to directly quantify dislocation density or microcrack density for use in a boundary value continuum mechanics problem. Because elasticity is directly influenced by damage since the number of atomic bonds responsible for elasticity decreases with damage (Basaran and Jiang, 2002). For an actual engineering system, it is extremely difficult for the current state of material science to provide the level of theoretical guidance that is needed to develop a predictive model based solely on dislocation or crack density considerations (Lemaitre, 1990). Instead measurement of degradation of global mechanical properties, such as elastic modulus, can be used to represent evolution of dislocation density or microcrack density.

In this study, several actual electronic BGA packages were subjected to thermal cycling in a Super AGREE thermal chamber, and intrinsic elastic modulus degradation of the critical solder joints (corner) was measured by MTS Nanoindenter XP system Continuous Stiffness Measurement (CSM) method periodically. The elastic modulus degradation is considered to be directly related to macromaterial degradation under fatigue (Lemaitre, 1990). The elastic modulus degradation as a function of number of thermal cycles was compared with measured inelastic strain accumulation tendency of solder joints. Inelastic strain is considered to be related to fatigue damage evolution. Even though inelastic strain alone cannot be a damage metric, because it would violate Clausius–Duhem inequality of thermodynamics.

Numerous fatigue life prediction models based on load-drop criterion have been proposed (e.g. Solomon, 1986a,b, 1989). But load-drop criterion does not lend itself to be measured directly in electronic packaging solder joints under thermal cycling. Simply, it is not possible to measure the load-drop in BGA solder joints during thermal cycling. Instead, load-drop is always observed by isothermal fatigue testing. Then these isothermal fatigue testing curves are used to develop empirical relations for the number of cycles to failure for thermal cycling. In this paper we show that mechanical degradation and thermal degradation are quite different. In this work, load-drop for different inelastic strain ranges was measured by stress–strain hysteresis loops from strain-controlled cyclic shear testing on thin layer Pb/Sn solder joints. Elastic modulus degradation evolution under thermal cycling and load-drop under isothermal shear testing were compared. Because most thermal fatigue life prediction approaches used in electronics industry are based on isothermal load-drop concept, this comparison is important to validate accuracy of this approach.

It may seem tempting to obtain an apparent elastic modulus from isothermal hysteresis loops and compare it with thermal cycling elastic modulus degradation. But this comparison would not be accurate, because Basaran and Tang (2002) have shown that for Pb/Sn alloys elastic modulus can only be measured by ultrasonic testing or Nano-indentation because creep deformation dominates the strain at high homologous temperatures.

In order to predict the thermal fatigue life of a BGA solder joints using Coffin–Manson approach, inelastic strain range of solder joints under thermal cycling is needed. Usually the inelastic strain is calculated by finite element method. In this project, we measured the thermal cycling induced inelastic strain in solder joints by means of high sensitivity Moiré interferometry. Then we used the measured stabilized inelastic strain range to predict thermal fatigue life of solder joints using Coffin–Manson *S*–*N* curve. At the same time we thermally cycled a BGA package to get the actual fatigue life. However, when we compared the Coffin–Manson thermal fatigue life prediction result with our fatigue testing under thermal cycling, we observed that error introduced by Coffin–Manson can be as high as an order of magnitude.

Our observations in this project show that an intrinsic damage metric, such as elastic modulus degradation measured directly during thermal fatigue testing instead of isothermal mechanical fatigue testing, is essential for thermal fatigue life prediction modeling. In other words if one wants to use Coffin–Manson type empirical relation for thermal cycling fatigue life, then the Coffin–Manson *S*–*N* curve must be constructed from thermal cycling not from mechanical testing. The difference is probably due to the thermal effects where at elevated temperatures Young's modulus for solder decrease significantly as a result it becomes very viscous and does not experience as much shear stress. Another reason could be the fact that temperature speeds up the microstructural evolution process which leads to equaixed structure which is known to have more fatigue resistance (Kashyap and Murty, 1981). Wen and Keer (2001) have shown that competing mechanism occur at elevated temperature. Dislocations occur easier with the help of thermal energy at the time annihilation of dislocation and recovery (healing) happens with thermal energy.

2. Experimental part

Two actual electronic BGA packages with Pb/Sn solder balls are used as specimens for thermal cycling. Fig. 1 shows the cross-section of the BGA module. FR-4 printed circuit board (PCB) and polymer connector layer are bounded by an array of eutectic Pb37/Sn63 solder joints. This study is focused on these solder joints because of the high CTE mismatch between the FR-4 PCB and the directional polymer connecter.

The BGA package was subjected to 2000 thermal cycles in a Super AGREE thermal chamber. The thermal loading profile is illustrated in Fig. 2. Each thermal cycles is 42 min long with 15 min dwelling on hot and cold sides. During thermal testing the package is fixed at the both ends of the

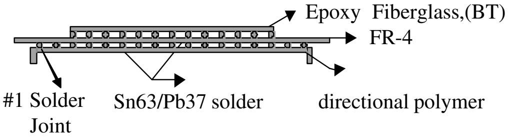

Fig. 1. Cross-section of BGA package.

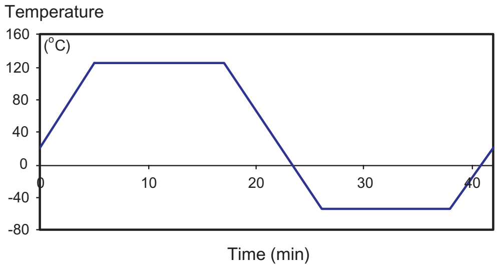

Fig. 2. The thermal loading profile of one cycle.

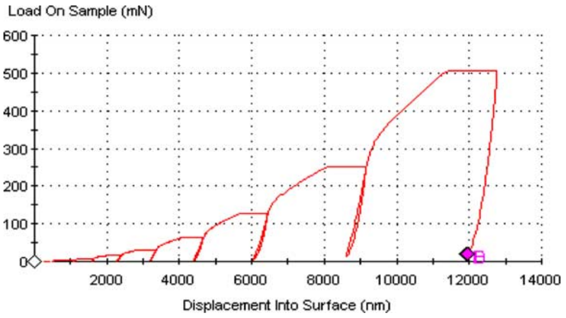

Fig. 3. Load–displacement of indentation for solder joints after 2000 thermal cycles.

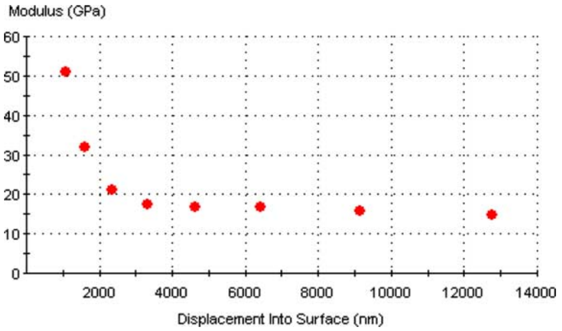

Fig. 4. Elastic modulus–displacement of indentation for solder joints after 2000 thermal cycles.

middle FR-4 PCB layer to simulate the actual boundary conditions in service. Thermally induced shear strain in solder joints due to the CTE mismatch between FR-4 PCB and polymer layer are cyclic in nature, and they result in thermomechanical fatigue of solder joints. During thermal fatigue testing, the critical part is always observed in solder joint #1 as shown in Fig. 1 (Zhao et al., 1999; Basaran et al., 2001; Zhao et al., 2000). Specimens were taken out of the thermal chamber periodically to measure elastic modulus degradation of solder joint #1.

MTS Nano-indenter XP system CSM option was used to measure the intrinsic elastic modulus degradation of solder joints. In fact, the solder joints are too small (500 μm in height) to be tested by any traditional testing methods to determine Young's modulus. The initial elastic modulus was measured before thermal cycling. During thermal cycling, the BGA module was taken out every 500 cycles to measure the elastic modulus. During testing load-controlled CSM method was used. Strain rate in CMS method is very high around 0.2/s. MTS Nano-indenter XP system CSM method yields the same elastic modulus values as ultrasonic testing and single crystal elasticity computations (Basaran and Tang, 2002). Due to size effect, usually the elastic modulus is larger when Nano-indenter tip is at the surface of specimen, and it decreases to a stable value after Nano-indenter reaches certain depth into the material. Size effect mechanics of Nano-indentation is outside the scope of this paper. An excellent review of the latter subject is presented in Begley and Hutchinson (1998) and Basaran and Tang (2002). Figs. 3 and 4 show Nano-indentation load–displacement and modulus–displacement of solder after 2000 thermal cycles.

The measured elastic modulus degradations of a solder joint for the two separate BGA package specimens under thermal cycling are shown in Fig. 5.

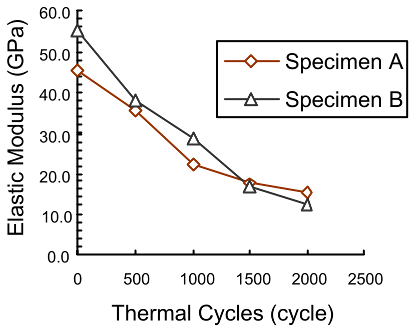

Fig. 5. Elastic modulus degradation of solder joint #1 under thermal cycling.

Elastic modulus can directly be calculated from single crystal elasticity, which is directly related to atomic lattice strength (Basaran and Jiang, 2002). Hence elastic modulus degradation directly reflects material microstructure degradation to macrolevel behavior. Using elastic modulus degradation as a damage metric, is very well established in the continuum damage mechanics literature (Kachanov, 1986; Lemaitre, 1990).

$$D = 1 - \frac{E_i}{E_0} \tag{1}$$

where D is damage state variable; E_0 is the initial elastic modulus; E_i is the elastic modulus at any point. Initially $D = 0$ and for complete failure $D = 1$. Fig. 6 shows damage evolution in a BGA solder joint as defined by Eq. (1) during thermal cycling. At the end of 2000 thermal cycles no crack initiation was observed in the solder joints by scanning electronic microscope (SEM).

Numerous fatigue prediction models consider inelastic strain accumulation alone as damage evolution criterion, such as Krajcinovic (1989) and Lemaitre (1990). Fig. 7 shows the measured inelastic strain accumulation in the same solder joint under thermal cycling by Moiré interferometry (Zhao et al., 1999). The behavior of elastic modulus degradation is quite different than inelastic strain accumulation trend. Studying Fig. 7 indicates that the inelastic strain increases vary fast during the first 20 thermal cycles, but inelastic strain growth becomes very slow after 20th thermal cycle. If inelastic strain is used as a sole damage criterion, that means fatigue damage evolves very fast within first 20 thermal cycles, then each thermal cycle just brings very little additional damage after 20 thermal cycles. However, the measured elastic modulus degradation up to 500 thermal cycles exhibit almost linear contribution to total material damage evolution, then after 1000 cycles slop reduces. It exhibits a mild slop reduction in Young's modulus verse number of cycles curve continuously. Yet it dose not exhibit the sharp curve we see in plastic strain accumulation.

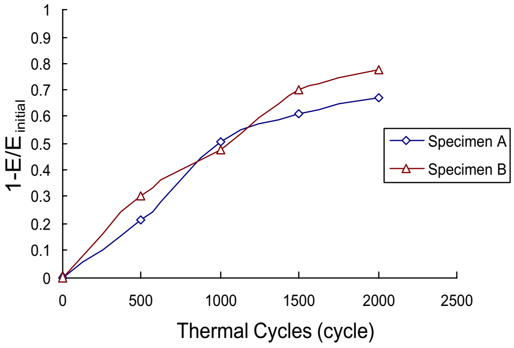

Fig. 6. Damage evolution as a function of number of thermal cycles.

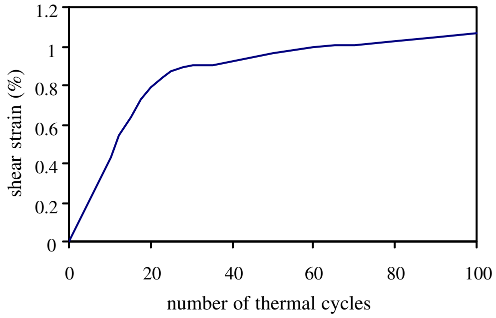

Fig. 7. Inelastic strain accumulation of solder joint #1 under thermal cycling.

Inelastic strain accumulation alone cannot appropriately reflect the physical mechanism of fatigue damage. Fatigue life prediction or damage evolution modeling that only consider inelastic strain range as fatigue criterion, violates Clausius–Duhem inequality. Basaran and Yan (1998) and Basaran and Tang (2002) have shown with extensive testing validations that entropy which is a measure of energy unavailable for work is a better metric for thermal fatigue damage evolution.

Traditionally, fatigue life prediction is based on Coffin–Manson *S*–*N* curves that are obtained from isothermal mechanical fatigue testing conducted on bulk samples. In this study, isothermal mechanical cyclic shear testing was also performed on a thin layer of (450 μm thick) Pb37/Sn63 solder alloy. The specimen is shown in Fig. 8.

MTS 858 testing system with hydraulic grips was used for strain-controlled cyclic testing. Fixture and load-train stiffness was measured with a benchmark specimen, and its effect was deducted to measure the response of the solder joints. The

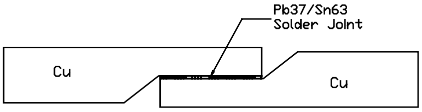

Fig. 8. Specimen for fatigue shear testing for Pb37/Sn63 thin layer solder joints.

fixture and load-train stiffness is determined as 0.0057 μm/N. The fatigue testing was conducted for different strain ranges at the strain rate of 10^{-4}/s. This strain rate was computed to match the approximate strain rate in our thermal cycling. Figs. 9–14 show strain–stress hysteresis loops for fatigue shear testing with strain ranges from 0.0026 to 0.04 at room temperature.

Traditionally used load-drop criterion is defined by the following equation (Solomon, 1989):

$$\phi = 1 - \frac{\Delta P}{\Delta P_m} \tag{2}$$

where ΔP is the load range at any point in the test and ΔP_m is the maximum load range which is measured in the first cycle or at most in the first few cycles. Load-drop ϕ varies from 0 at the start of the test to 1 when complete failure has occurred and no load can be supported. Load-drops corresponding to Figs. 9–14 are illustrated in Fig. 15. From the hysteresis loops of fatigue shear testing, it can be found that there is small load-drop from the start of testing to one hundred or several thousand cycles, depending on different strain

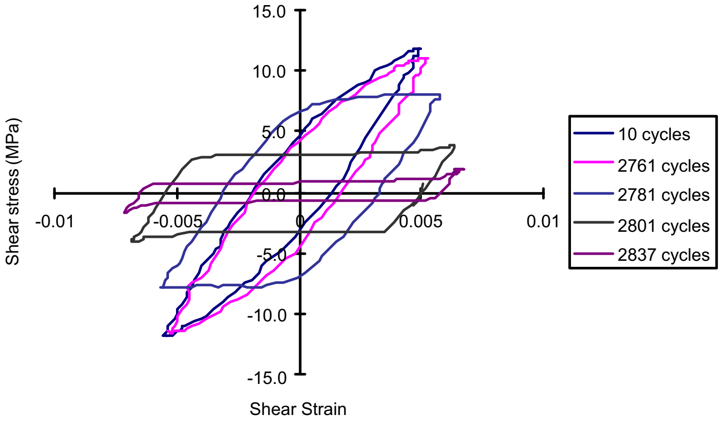

Fig. 9. Shear strain–stress hysteresis loops for solder joints with inelastic strain range of 0.0026, strain rate 1.67×10^{-4}/s, 22 °C.

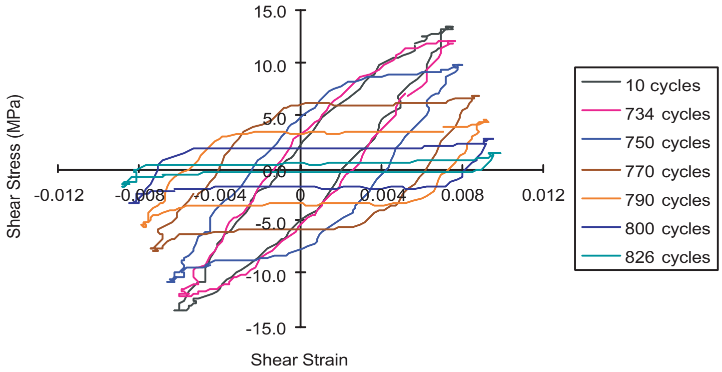

Fig. 10. Shear strain–stress hysteresis loops for solder joints with inelastic strain range of 0.004, strain rate 1.67×10^{-4}/s, 22 °C.

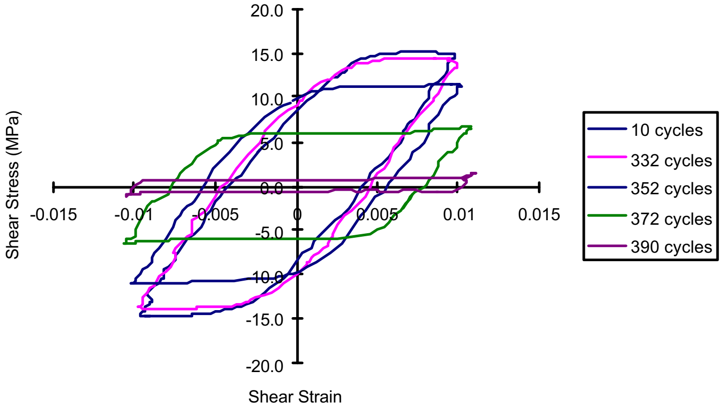

Fig. 11. Shear strain–stress hysteresis loops for solder joints with inelastic strain range of 0.008, strain rate 1.67×10^{-4}/s, 22 °C.

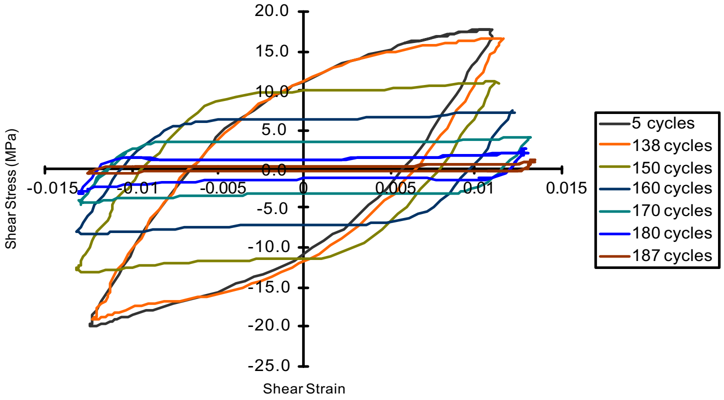

Fig. 12. Shear strain–stress hysteresis loops for solder joints with inelastic strain range of 0.012, strain rate 1.67×10^{-4}/s, 22 °C.

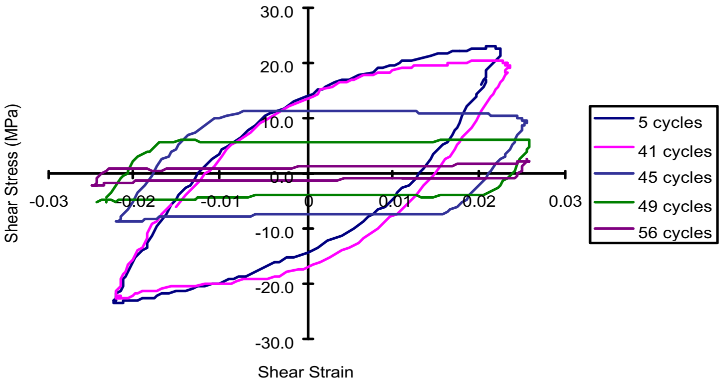

Fig. 13. Shear strain–stress hysteresis loops for solder joints with inelastic strain range of 0.022, strain rate 1.67×10^{-4}/s, 22 °C.

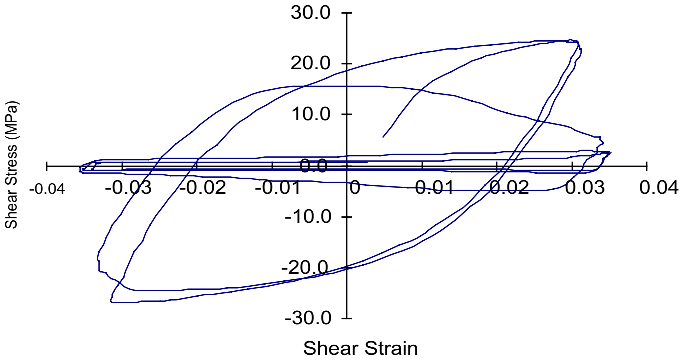

Fig. 14. Shear strain–stress hysteresis loops for solder joints with inelastic strain range of 0.04, strain rate 1.67×10^{-4}/s, 22 °C.

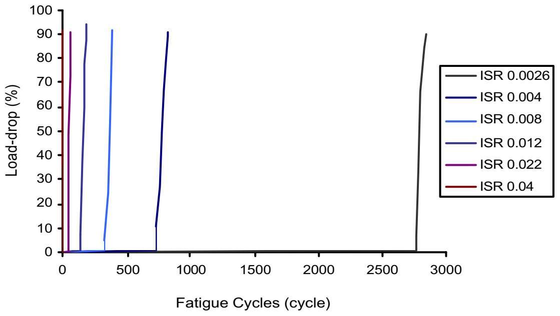

Fig. 15. Load-drop ϕ vs. fatigue cycles for different inelastic strain range (ISR).

range. After a certain critical loading cycle is reached, load-drop becomes much bigger and then increases very quickly until complete failure happens. This behavior has been explained by percolation damage mechanics theory by several investigators such as Wen and Keer (2001). This finding also supports our microscopy observation. We did not see any crack initiation until near the end of the testing. This behavior can be clearly observed from Fig. 15. For other strain rates and temperatures for which hysteresis loops are not shown, the same situation is also observed. Stolkarts et al. (1999), Liang et al. (1996) and Solomon (1989) reported observing the same behavior on bulk solder specimens.

Nevertheless, the damage evolution tendency of isothermal fatigue shear testing based on load-drop criterion is totally different from the material degradation under thermal fatigue testing based on elastic modulus degradation measurement. Isothermal load-drop evolution is also different from the tendency of inelastic strain accumulation of solder joints under thermal cycling.

It is also known that phase growth (coarsening) is also used as damage metric in solder joints fatigue studies (Frear et al., 1994). Coarsening is considered to be detectable microstructure degradation characteristic in Pb/Sn solder joints. Isothermal load-drop behavior is also different from phase coarsening of solder joints under thermal cycling. Fig. 16 shows measured grain growth of the solder joints in the BGA package under thermal cycling (Basaran and Wen, 2002). When we compare the general behavior of Coarsening curve with load-drop evolution, they look very different. Coarsening happens in the first 100 thermal cycles and then stabilizes. Load-drop progress very slowly until a critical point is reached.

In fracture mechanical load-drop criterion is often used as continuous crack propagation met-

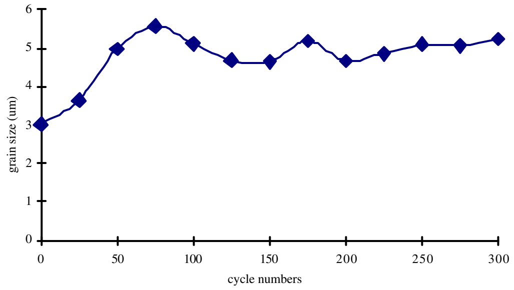

Fig. 16. Coarsening of solder joints under thermal cycling.

ric. Considering that crack growth itself is difficult to be measured during testing, load-drop can be used as a good measure of microcrack coalescence. As cracks increase in volume effective stress in the cross-section increases. After the critical cycle is reached, apparent load-drop can be observed, and from that point load-drop start increasing very fast. The critical cycles in our experiments were No. 2761st, 734th, 332nd, 138th, 41st and 3rd cycle in Figs. 9–14 respectively. During our testing we observed that a single macrocrack initiated at these loading cycles. In fact, before a macrocrack initiated, load-drop is too small to be observed, in spite of the fact that material degradation is progressively increasing with microcracks and void growth and coalescence, which is reflected by measured elastic modulus degradation.

Although the observation of load-drop during isothermal mechanical fatigue testing is well correlated with macrocrack initiation, load-drop criterion is not suitable for thermal fatigue prediction of solder joints in electronic package. Because for thermal fatigue prediction of solder joints in electronic package, the damage evolution prior to macrocrack initiation is important. In our 2000 thermal cycling BGA package solder joints did not fail or develop any microcrack that can been seen with SEM. Our SEM micrographs did not show any macrocrack initiation, yet elastic modulus degraded significantly. But load-drop criterion cannot cover this stage very effectively. This is probably due to, isothermal fatigue mechanism is quite different than thermal fatigue mechanism. In the following section we try to clarify this statement.

Currently the electronics industry uses Coffin–Manson *S–N* curve Eq. (3), for thermal fatigue life prediction.

$$N_f^{\alpha}\Delta\gamma_p = \theta \tag{3}$$

where N is the number of cycles to fatigue failure; $\Delta\gamma_p$ is plastic strain range; α is called the fatigue ductility exponent. It is the slope of *S–N* curve; θ is called the fatigue ductility coefficient. It is usually empirically obtained from isothermal mechanical fatigue testing using load-drop as a criterion (Frear et al., 1994). However, using Coffin–Manson *S–N* curve to predict thermal fatigue life of solder joints yields an error as high as an order of magnitude in the number of cycles to failure. It is questionable to assume that the approach based on isothermal fatigue data can be directly used to model the thermal fatigue process, ignoring the fact that the temperature has significant effects on dislocation dynamics, the material properties and hysteresis strain energy dissipation. From the experimental observation in this study, the thermal fatigue damage mechanism for solder joints in electronic package cannot be appropriately reflected by load-drop criterion of isothermal mechanical fatigue testing. Fig. 17 shows logarithmic Coffin–Manson *S–N* curve for Pb37/Sn63 solder alloy at 22 °C and strain rate 0.167/s, which is obtained from our isothermal fatigue shear test data of thin layer solder joints with fatigue failure definition of 90% load-drop. Using this *S–N* curve, the fatigue life corresponding to 0.01 inelastic strain range can be determined to be around 200 cycles. Inelastic strain range of solder joints in BGA package for each thermal cycle, after 100 cycles, is little more than 0.01, according to Moiré interferometry measurements in Fig. 7. Using the *S–N* curve in Fig. 16, we can predict that the solder joints would fail within less than 300 thermal cycles. But the failure was not observed even after several thousand thermal cycles in thermal fatigue testing of actual electronic package. The reason is that macrocrack in solder joints did not initiate even after several thousand thermal cycles which we observed with SEM. Load-drop criterion, which is suitable to describe macrocrack

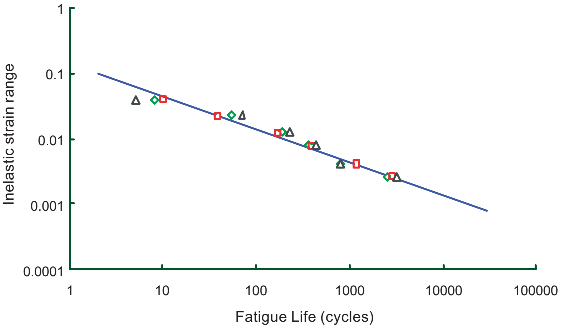

Fig. 17. Logarithmic inelastic strain range vs. fatigue life for Pb37/Sn63.

propagation, cannot accurately describe the damage mechanism of solder joints under thermal fatigue loading.

Solder joints under thermal cycling exhibit different failure mechanism than solder joints of same scale under mechanical fatigue loading. Fig. 18 shows a failed solder specimen with a macrocrack under mechanical shear cycling. Fig. 19 shows a solder joints in a BGA module after 1500 thermal cycles. No macrocrack can be observed. But in fact, elastic modulus value has degraded significantly under thermal fatigue loading.

Hence based on our observations in this study we can deduce that in order to make accurate thermal fatigue life predictions for solder joints in electronic packaging, a damage evolution model directly based on thermal fatigue testing is necessary. Conventional approach based on isothermal fatigue testing is not appropriate for thermal fatigue life prediction. It is obvious that damage mechanism under isothermal mechanical fatigue loading and thermal fatigue loading evolve quite differently.

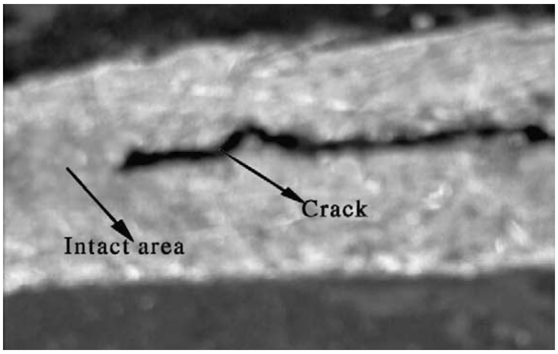

Fig. 18. Crack in solder joints layer under fatigue shear loading.

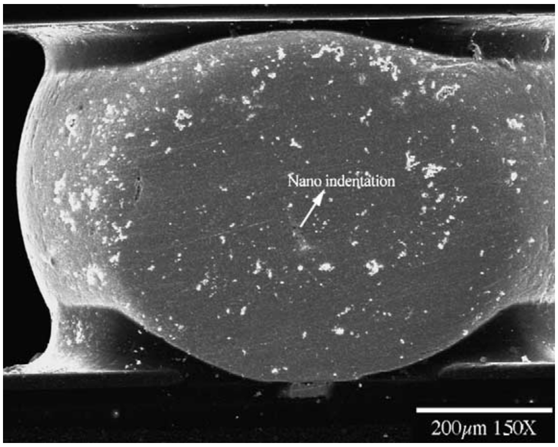

Fig. 19. Solder joints in BGA module after 1500 thermal cycles.

3. Conclusions

Traditional load-drop evolution of isothermal fatigue testing is significantly different from evolution of detectable thermal fatigue damage quantities, such as elastic modulus degradation, inelastic strain accumulation and microstructure phase coarsening. Fatigue life prediction based on isothermal fatigue test data, such as Coffin–Manson fatigue equation, cannot be appropriately used for thermal fatigue life prediction. Error introduced by these approaches can be as high as orders of magnitude. Approach directly developed from thermal fatigue testing is necessary for accurately predicting thermal fatigue.

Using inelastic strain accumulation alone as a damage metric would lead to inaccurate damage quantification. Detectable global modulus degradation, such as elastic modulus degradation, is directly related to microstructural damage, should be used for thermal fatigue damage evolution modeling. Phase coarsening is not a reliable damage metric.

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