Experimental verification of improvement of phase shifting moiré interferometry using wavelet-based image processing

Heng Liu
Alexander N. Cartwright, MEMBER SPIE
Cemal Basaran
University at Buffalo
The State University of New York
Electronic Packaging Laboratory
Buffalo, New York 14260
E-mail: anc@eng.buffalo.edu

Abstract. Phase shifting interferometry is combined with wavelet-based image processing techniques to extract precise phase information for applications of moiré interferometry. Specifically, a diffraction grating identical to the specimen grating is used to introduce the additional phase shifts needed to implement phase shifting moiré interferometry. The phase map is calculated with the four-step phase shifting algorithm with 90-deg relative shifts between adjacent frames. Subsequently, continuous wavelet transform processing is applied to the resultant in-plane strain map to eliminate the noise generated through the derivative calculation. To demonstrate the usefulness of this technique, uniform vertical loading is applied to a three-layer ball grid array (BGA) package and the strain is experimentally obtained. The reconstructed phase and in-plane strains are compared to the simulation results obtained using Ansys®.

Subject terms: moiré interferometry; phase shifting interferometry; ball grid array packaging; continuous wavelet transform; discrete wavelet transform; surface mount technology.

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1 Introduction

Over the last few decades, the electronic packaging industry has rapidly developed. Packaging based on surface mount technology (SMT), including ball grid array (BGA) packaging, plastic ball grid array (PBGA) packaging, ceramic ball grid array (CBGA) packaging, fine ball grid array (FBGA) packaging, and chip scale packaging (CSP), has become dominant in electronic packaging design. With the shrinking size of the packages and increasing number of components integrated into the packages, the reliability of the packages becomes one of the biggest concerns in the packaging development process. Due to the sandwich structure of these SMT packages, the accurate measurement of the in-plane deformation is essential to reliability studies of these electronic packages. Moiré interferometry, as an inspection tool with high sensitivity and the ability for whole-field measurement, is widely used by many research groups to investigate the reliability of these packages under thermal and vibration loading.¹⁻⁵

In moiré interferometry, a cross-line diffraction grating is transferred to the cross section of the specimen that is the area of interest in the reliability study. The grating attached to the specimen surface is usually referred to as the specimen grating. The specimen grating diffracts the incident laser beams, and the diffracted beams form the interferometry fringe patterns. The two horizontal laser beams measure the horizontal deformation of the specimen, called the U field, and the two vertical beams measure the vertical deformation of the specimen, called the V field. Fringe analysis is then applied to the resultant U-field and V-field fringe patterns to extract the in-plane deformation and in-plane strains. The general relationship between the moiré fringe and the in-plane strain is given by¹:

\[ \varepsilon_x = \frac{1}{f} \frac{\partial N_u}{\partial x}, \]
\[ \varepsilon_y = \frac{1}{f} \frac{\partial N_v}{\partial y}, \]
\[ \gamma_{xy} = \frac{1}{f} \left( \frac{\partial N_u}{\partial y} + \frac{\partial N_v}{\partial x} \right), \]  

where \( f \) is the spatial frequency of the specimen grating, \( N_u \), \( N_v \) are the fringe orders of the U-field and V-field interferograms, \( \varepsilon_x \), \( \varepsilon_y \) are the in-plane normal strains, respectively, and \( \gamma_{xy} \) is the shear strain. The most commonly used specimen grating has a groove density of 1200 lines/mm, which leads to a deformation sensitivity of 0.417 \( \mu \text{m} \)/fringe.

Phase shifting interferometry (PSI) was originally proposed by Carre in 1966.⁶ Theoretically, phase shifting interferometry allows us to reconstruct the phase information

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wrapped in the fringe patterns. Phase shifting moiré has been developed since 1992. However, the reconstructed phase map obtained in these measurements usually suffers from the background noise due to imperfect grating replication (such as speckles) and defects on the reflection aluminum layer, especially for a relatively large area of interest. To utilize the phase map to obtain pixel-by-pixel in-plane strain information, the phase map from phase shifting moiré interferometry needs to be processed. In this study, a wavelet-based image processing algorithm is proposed and discussed to eliminate the associated noise. The result obtained using the image processing algorithm on images obtained from the mechanical loading phase shifting moiré interferometry (PSMI) experiment is verified by an Ansys® simulation (a general purpose finite element program, Canonsburg, PA) simulation of the experiment.

In Sec. 2, PSMI is briefly presented. The necessary wavelet-based image processing methods are then explored in Sec. 3. Sections 4 and 5 describe the experimental setup, the results from the experiment, and the results from the Ansys® simulation.

2 Phase Shift Moiré Interferometry

It is well known that an interferogram can be generally expressed as

\[ I = I_0 \cdot (1 + \gamma \cdot \cos \varphi), \]  

where \( I \) is the interference density, \( I_0 \) is the background illumination, \( \gamma \) is the fringe visibility, and \( \varphi \) is the phase difference of the two interfering laser beams. In fringe analysis, each fringe represents either the darkest or the brightest interference intensity, depending on how we define the fringe. In either definition, each fringe corresponds to a \( 2\pi \) phase change.

In moiré interferometry, the phase \( \varphi \) is proportional to the in-plane deformation of the specimen. Therefore, the desired information from the interferogram analysis is the reconstruction of the phase information that is derived through the fringe analysis. Since there are three unknowns in Eq. (2), \( I_0, \gamma, \) and \( \varphi, \) \( n \) interferometry frames (\( n \geq 3 \)) can be used to solve the \( n \) corresponding equations to accurately obtain the phase \( \varphi \). In general, to reduce error, at least one redundant interferogram is taken. Equation (3) presents a four-step phase shifting algorithm:

\[ I_1 = I_0 + I_0 \gamma \cos(\varphi), \]
\[ I_2 = I_0 + I_0 \gamma \cos(\varphi + \frac{\pi}{2}), \]
\[ I_3 = I_0 + I_0 \gamma \cos(\varphi + \pi), \]
\[ I_4 = I_0 + I_0 \gamma \cos(\varphi + \frac{3\pi}{2}). \]  

According to Eq. (3), the additional phase shifts of \( \pi/2, \pi, \) and \( 3\pi/2 \) are introduced to the second, third, and fourth interferograms with respect to the first interferogram. Therefore, the phase information can be derived as a function of the intensity of the four interferograms as:

\[ \tan \varphi = \frac{I_4 - I_2}{I_1 - I_3}. \]  

Once \( \varphi \) is determined, the in-plane strains can be calculated as the derivatives of the phase map:

\[ \varepsilon_x = \frac{1}{2\pi f} \frac{\partial \varphi_x}{\partial x}, \]
\[ \varepsilon_y = \frac{1}{2\pi f} \frac{\partial \varphi_y}{\partial y}, \]
\[ \gamma_{xy} = \frac{1}{2\pi f} \left( \frac{\partial \varphi_x}{\partial y} + \frac{\partial \varphi_y}{\partial x} \right). \]  

There are many methods that can be employed to introduce additional phase shifts. This includes translating mirrors, rotating waveplates, translating gratings, and electro-optical phase modulators. In this study, a translating diffraction grating is used to provide the phase shifting, as shown in Fig. 1. The translating diffraction grating has exactly the same profile as the specimen grating, a sinusoidal phase grating. The reflection function of the sinusoidal phase grating is given by:

\[ R(x_0) = \exp\{i[b_0 + b_1 \sin(\Omega x_0)]\}. \]
where \( x_0 \) denotes the coordinate of the grating profile and \( \Omega \) is the grating frequency, i.e., \( 2\pi \) over the grating period. Since the cross-line diffraction gratings are symmetrical in the \( x_0 \) and \( y_0 \) direction, the conclusion from a 1-D grating analysis can be readily extended to a 2-D grating. For simplicity, the analysis is presented for a single dimension \( (x) \) in Eqs. (6) through (10). The translated grating gives the translated reflection function as:

\[
R'(x_0) = \exp\{i[b_0 + b_1 \sin(\Omega x_0 + \delta)]\},
\]

where \( \delta \) is the phase term of the corresponding translation. Utilizing the Fresnel diffraction integral to analyze the diffraction field of the sinusoidal grating, we obtain

\[
U_R(x_0) \propto \int \{A \exp[jb_0 + jb_1 \sin(\Omega x_0)] \times \exp[jkx_0 \sin \theta]dx_0\},
\]

where \( U_R(x_0) \) is the diffraction field.

Recalling the Jacobi-Anger expansion,

\[
\exp(ib \sin \phi) = \sum_{m=-\infty}^{\infty} [J_m(b) \exp(im\phi)],
\]

where \( J_m(b) \) is the Bessel function of the first kind, it is easy to obtain the relationship between the diffraction fields of the translated grating and the original grating:

\[
U_R'(x_0) = U_R(x_0) \exp(i\delta).
\]

Note that only first-order diffraction is considered in Eq. (10), since we assume that other orders are either very weak or nonexistent.

As shown in Fig. 1, the translating grating also serves as a beamsplitting device. Therefore, the translation \( d \) of the grating will introduce a \( \pm 2\pi d/\Lambda \) phase shift in both beams, where \( \Lambda \) is the grating period. Thus, the total phase shift introduced to the interferogram is \( 4\pi d/\Lambda \). In other words, to achieve a \( \pi/2 \) phase shift, a translation of \( \Lambda/8 \) is required. In the case of a 1200-lines/mm grating, this corresponds to a 0.104-\( \mu \)m translation.

As we discussed in Sec. 1, although the phase shifting technique is able to provide a precise phase map of the interferogram, the phase calculated from Eq. (4) is usually noisy because of the imperfect nature of experimental techniques. In addition to the random intensity noise as a result of laser power fluctuation, defects in optics and imperfect grating replication are also important noise sources. More importantly, the traditionally used fast Fourier transform (FFT) filtering is not effective at eliminating the noise, as the interferogram fringes cover a wide range of spatial frequencies. Basically, this is because the FFT only maintains the spectrum of the entire signal, but there is no link between the spectrum and the local signal. Therefore, when the fringe or phase information in the interferogram covers a wide range of spatial frequency, especially when the spatial frequency of the local noise in one region is incompatible with the spatial frequency of the signal in another region, we cannot rely on FFT to obtain reliable filtering results. Fortunately, a wavelet transform can be used to overcome the disadvantages of the Fourier transform in fringe analysis.12–20 The biggest difference between Fourier analysis and wavelet analysis is that the latter utilizes a pulsed function and the former utilizes a continuous function when filtering the signal. Therefore, the latter will maintain the characteristic of the local signal and the former will lose that localized information. The wavelet analysis includes continuous wavelet transform (CWT) and discrete wavelet transform (DWT). In wavelet analysis, the signal is compared with a wavelet that is scaled from a pulse-like (which only exists for a very short period of time) mother wavelet. The scaling of the mother wavelet provides a wide frequency range for the analysis of the local signal. Basically, in CWT, the mother wavelet is scaled by a wide range of scale parameters to filter the signal; in DWT, the mother wavelet is scaled by \( 2^N \) (\( N \) is the integer) to filter the signal. In this study, CWT is used to maintain fiber is used to deliver the laser beam to the moiré optics while maintaining the polarization state of the laser beams.

### 3 Wavelet-Based Image Processing

As we discussed in Sec. 1, although the phase shifting technique is able to provide a precise phase map of the interferogram, the phase calculated from Eq. (4) is usually noisy because of the imperfect nature of experimental techniques. In addition to the random intensity noise as a result of laser power fluctuation, defects in optics and imperfect grating replication are also important noise sources. More importantly, the traditionally used fast Fourier transform (FFT) filtering is not effective at eliminating the noise, as the interferogram fringes cover a wide range of spatial frequencies. Basically, this is because the FFT only maintains the spectrum of the entire signal, but there is no link between the spectrum and the local signal. Therefore, when the fringe or phase information in the interferogram covers a wide range of spatial frequency, especially when the spatial frequency of the local noise in one region is incompatible with the spatial frequency of the signal in another region, we cannot rely on FFT to obtain reliable filtering results. Fortunately, a wavelet transform can be used to overcome the disadvantages of the Fourier transform in fringe analysis.12–20 The biggest difference between Fourier analysis and wavelet analysis is that the latter utilizes a pulsed function and the former utilizes a continuous function when filtering the signal. Therefore, the latter will maintain the characteristic of the local signal and the former will lose that localized information. The wavelet analysis includes continuous wavelet transform (CWT) and discrete wavelet transform (DWT). In wavelet analysis, the signal is compared with a wavelet that is scaled from a pulse-like (which only exists for a very short period of time) mother wavelet. The scaling of the mother wavelet provides a wide frequency range for the analysis of the local signal. Basically, in CWT, the mother wavelet is scaled by a wide range of scale parameters to filter the signal; in DWT, the mother wavelet is scaled by \( 2^N \) (\( N \) is the integer) to filter the signal. In this study, CWT is used to
eliminate the localized image noise and extract a smooth phase map. However, a smooth reconstructed phase does not necessarily guarantee that the first derivative of the phase is also smooth. Therefore, to reliably obtain in-plane strain that is proportional to the first derivative of the phase, further curve smoothing is required. A curve-smoothing algorithm based on DWT is used to capture the approximation of the phase derivative to obtain reliable in-plane strain distribution. Both CWT and DWT are discussed in this section.

3.1 Continuous Wavelet Transform

Comparable to FFT analysis, in which a continuous function \( \exp(jot) \) is convolved with the signal to achieve the Fourier spectrum of the signal, CWT can be understood as the convolution of the signal and a pulse-like wavelet function:

\[
S(a,b) = \frac{1}{a} \int_{-\infty}^{\infty} s(t) \cdot M \left( \frac{t-b}{a} \right) \cdot db,
\]

where \( M(t) \) is the mother wavelet, \( a \) is the scaling parameter that controls the scale of the mother wavelet, \( b \) is the shifting parameter to ensure that the convolution covers the entire sequence of the signal, \( s(t) \) is the signal, and \( S(a,b) \) is the CWT coefficient. The CWT coefficient \( S(a,b) \) is a 2-D function for a 1-D signal \( s(t) \).

Since the signal to be processed in this study, the interference signal, has the format of a sinusoidal function, the Morlet mother wavelet is selected:

\[
M(x) = \exp\left(-\frac{x^2}{2}\right) \cdot \exp(j \cdot \omega_0 \cdot x).
\]

The pulse shape of the Morlet wavelet is ensured by the Gaussian factor and the sinusoidal characteristic of the interference signal will be captured by the complex exponential factor. \( \omega_0 = 2\pi \) is used in this study. By substituting Eqs. (1) and (12) into Eq. (11), we obtain the CWT of the interference signal as:

\[
S(a,b) = I_0 \cdot \sqrt{2\pi} \cdot \exp\left(-\frac{\omega_0^2 a^2}{2}\right) + \frac{1}{2} \cdot I_0 \cdot \gamma \cdot \sqrt{2\pi} \cdot \exp\left(-\frac{(\omega_0 + a \cdot \omega_s)^2}{2}\right) \cdot \exp(-j \cdot \omega_s \cdot b) + \frac{1}{2} \cdot I_0 \cdot \gamma \cdot \sqrt{2\pi} \cdot \exp\left(-\frac{(\omega_0 - a \cdot \omega_s)^2}{2}\right) \cdot \exp(j \cdot \omega_s \cdot b).
\]

(13)

Clearly, the first term is negligible due to the decaying characteristic of the Gaussian function. Furthermore, the scaling parameter \( a \) is chosen to be positive. Therefore, depending on the sign of the spatial signal frequency \( \omega_s \), either the second term or the third term in Eq. (13) is negligible. The CWT coefficients of the interference signal are thus reduced to

\[
S(a,b) = \frac{1}{2} \cdot I_0 \cdot \gamma \cdot \sqrt{2\pi} \cdot \exp\left(-\frac{(\omega_0 \pm a \cdot \omega_s)^2}{2}\right) \cdot \exp(\mp j \cdot \omega_s \cdot b).
\]

(14)

Close inspection of Eq. (14) shows that for various scaling parameters \( a \), the coefficients are maximized at \( a_p = \pm \omega_0 / \omega_s \). The \( a_p \) array for all \( b \) positions forms the so-called “ridge” of the CWT coefficients. At the ridge, the CWT coefficient satisfies:

\[
S(a_p,b) = \frac{1}{2} \cdot I_0 \cdot \gamma \cdot \sqrt{2\pi} \cdot \exp(\mp j \cdot \omega_s \cdot b).
\]

(15)

This is how the actual local signal frequency is extracted using CWT filtering. The noise-induced frequency components, since they show a small magnitude in the CWT coefficients map, are eliminated. From Eq. (15), it is clear that the phase of the CWT ridge is exactly the same as the phase of the interferogram, and thus can be used to reconstruct the phase information from the signal.

3.2 Carrier Phase

One limitation of CWT filtering in an actual application is the finite data sequence. Such limitation can be successfully compensated by the introduction of a carrier phase in the CWT filtering. The finite data array gives rise to the revised CWT coefficient

\[
S(a,b) = \exp\left(-\frac{(\omega_0 - a \cdot \omega_s)^2}{2}\right) \cdot \exp(-j \cdot \omega_s \cdot b) \times \left(\frac{1}{a} \int_{b-D}^{b+p} \exp\left(-\frac{(t-a)^2}{2a^2}\right) \cdot dt \right).
\]

(16)

where \( a = b - ja_0(\omega_0 - a \cdot \omega_s) \) and \( 2D \) is the length of the finite data array. When \( D \approx T_s \) (the period of the signal), the finite data effect can be neglected, i.e., Eq. (16) can be accurately approximated by Eq. (13), and we can perform the ridge detection to complete the phase reconstruction. However, when \( D \) is close to \( T_s \), for example, one or less than one period of the signal is included in the available dataset, the difference between Eqs. (13) and (16) cannot be ignored. The ridge obtained by evaluating the maximum magnitude of the CWT coefficient \( S(a,b) \) will result in an incorrect phase reconstruction.

To solve this problem of CWT filtering, the concept of a carrier phase is introduced. Without the addition of the carrier phase, the input of the CWT filtering process is

\[
I = \cos(\varphi),
\]

(17)

where \( \varphi \) is the phase calculated from the phase shifting interferograms. To overcome the CWT problem with sparse fringes, a computer-generated carrier phase \( \theta_c \) is added to the phase \( \varphi \). The input equation is thus changed to

\[
I = \cos(\varphi + \theta_c).
\]

(18)

Clearly, with the addition of the carrier phase, the spatial frequency of the signal that is fed into the CWT process is
modified. \( \theta_c \) is selected so that the total input period is small enough to neglect the finite length of the data array. Ridge detection is then employed to search for the local maximum of the CWT coefficients. By subtracting the carrier phase from the unwrapped angle of the ridge, the filtered phase of the phase shifting calculation is recovered. The finite data length is no longer a problem. Therefore, the CWT filtering process we propose can be used for interferograms with virtually any type of fringe distribution.

3.3 Discrete Wavelet Transform

The CWT reconstructed phase map can be successfully obtained. However, a smooth phase map does not necessarily give rise to an equally smooth derivative, which is required to obtain the in-plane strain distribution. A discrete wavelet transform is used in this study to capture the low-frequency outline of the noisy derivative. The fundamental difference between CWT and DWT is that the former utilizes a wide range of scaling parameter \( a \) to capture the local frequency of the signal, and the latter utilizes scales that are a power of 2 only to analyze the signal. The diagram of DWT multirate analysis \(^{21}\) is shown in Fig. 4. Figure 4(a) shows the decomposition (also called analysis) part, and Fig. 4(b) shows the reconstruction (also called synthesis) part.

The input signal \( x(n) \) is filtered with a lowpass filter \( h_1(n) \) and highpass filter \( h_2(n) \). Both filters are carefully designed wavelet filters. The results are called approximation \( a_1(n) \) and detail \( d_1(n) \), respectively. \( a_1(n) \) is then downsampled by the rate of 2 and fed into the second stage of the \( h_1(n), h_2(n) \) filter pair to obtain the second-level approximation \( a_2(n) \) and detail \( d_2(n) \). Iterative filtering and downsampling compose the decomposition part of the DWT analysis. Similarly, iterative upsampling and filtering are used in the corresponding reconstruction part of DWT to exactly reconstruct the input \( x(n) \) as \( y(n) \). Note that \( h_1(n), h_2(n), g_1(n), \) and \( g_2(n) \) are carefully designed filter banks. \(^{22}\) The filter bank is illustrated by

\[
\begin{align*}
G_1\{\exp[j(\pi - \omega)]\}G_1\{\exp[j\omega]\} &= 0, \\
G_1\{\exp[-j\omega]\} &= -G_1\{\exp[j\omega]\}, \\
G_2\{\exp[j\omega]\} &= G_1\{-\exp[j\omega]\}, \\
H_1\{\exp[j\omega]\} &= G_1\{\exp(-j\omega)\}, \\
H_2\{\exp[j\omega]\} &= G_2\{\exp(-j\omega)\}.
\end{align*}
\]

The filter bank ensures the perfect reconstruction for each two-channel decomposition/reconstruction filtering stage, thus the final reconstruction \( y(n) \) should be the perfect reconstruction of input \( x(n) \). In this study, since the purpose is to remove high-frequency noise of the derivative of the phase map, we only reconstruct the highest level approximation \( a_N(n) \), where \( N \) is the number of decomposition/reconstruction stage. It is easily proved that the resultant spectrum \( Y(\omega) \) is the lowest \((1/2^N)\) frequency part of the input spectrum \( X(\omega) \). \(^{23}\) For example, if only the second-level approximation \( a_2(n) \) is reconstructed, the resulting spectrum will be

\[
\begin{align*}
H_1(\omega) &\quad G_1(\omega) \\
H_2(\omega) &\quad G_2(\omega) \\
H_3(2\omega) &\quad G_1(2\omega) \\
H_3(2\omega) &\quad G_2(2\omega)
\end{align*}
\]
According to the approximate spectrum magnitude of the filter bank $H_1(\omega)$, $H_2(\omega)$, $G_1(\omega)$, and $G_2(\omega)$, as shown in Fig. 5, only the frequency part within $\omega \in [0, \pi/4]$ of $X(\omega)$ is transferred to $Y(\omega)$. This is how the DWT is utilized to smooth the in-plane strain calculation.

4 Experiment

To verify the process proposed in the preceding section, an experiment was conducted on an actual BGA package. The diagram of the BGA package is shown in Fig. 6. We define the layer numbers 1 to 3 from the bottom up. The first layer, i.e., the bottom layer, is a bismaleimide triazine (BT) substrate; the second layer, i.e., the middle layer, is an eutectic Pb/Sn solder alloy BGA encapsulated with underfill epoxy; and the third layer, i.e., the top layer, is the silicon die. The length of the specimen is about 20 mm and the width is 7 mm. The heights of the three layers are 1.32, 0.096, and 0.714 mm, respectively. A $1.13 \times 10^5$ N/m$^2$ uniformly distributed mechanical pressure is applied to the specimen at the top surface of the silicon die, as shown in Fig. 7. The purpose of applying such a small load is to make sure that each layer in the structure remains in the linear-elastic region. There is no plastic deformation introduced, so that measurements can easily be compared with simple elasticity solutions.

Uniform loading is then applied by a vertically aligned actuator. The actuator has a travel range of 50 mm and translation resolution of 0.016 mm. The maximum force that can be applied through the actuator is 120 N. The actual force applied is 15.84 N. The applied force is spread from the end of the actuator to the entire top surface of the silicon die by inserting an aluminum block between the actuator and the specimen. The size of the aluminum block is $20 \times 9.7$ mm. The resultant pressure on the specimen is therefore the division of the total force by the area of the top surface: $15.84/(0.02 \times 0.007) = 1.13 \times 10^5$ N/m$^2$. To prevent the potential lateral load of components, a thin layer of grease is applied between the aluminum block and the specimen. Ansys®, a general purpose finite element program, is used to perform structural analysis of the tested package. Elastic linear analysis is used due to the small pressure applied to the specimen. The material properties used in the simulation are described in Table 1. The simulation results are shown in Fig. 8. The symmetry of both the U and V displacements is readily observed.

![Mesh and Load](image1)

(a) Mesh and Load

![U displacement](image2)

(b) U displacement

![V Displacement](image3)

(c) V Displacement

![Simulation result by Ansys®](image4)

Fig. 8 Simulation result by Ansys®: (a) mesh and load, (b) U displacement, and (c) V displacement.

![U-field: initial](image5)

(a) $I_1 @ \delta_1=0^\circ$

(b) $I_2 @ \delta_2=90^\circ$

(c) $I_3 @ \delta_3=180^\circ$

(d) $I_4 @ \delta_4=270^\circ$

(e) $\varphi_{ai}$

Fig. 9 U-field: initial: (a) $I_1$ at $\delta_1=0$ deg, (b) $I_2$ at $\delta_2=90$ deg, (c) $I_3$ at $\delta_3=180$ deg, and (d) $I_4$ at $\delta_4=270$ deg.

### Table 1 Material parameters.

<table>
<thead>
<tr>
<th></th>
<th>$E$ (Young’s Modulus)</th>
<th>$\rho$ (Poisson Ratio)</th>
<th>Width</th>
<th>Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1 (BT)</td>
<td>$17.5 \times 10^9$ Pa</td>
<td>0.32</td>
<td>20 mm</td>
<td>1.32 mm</td>
</tr>
<tr>
<td>M2 (BGA)</td>
<td>$24.3 \times 10^9$ Pa</td>
<td>0.32</td>
<td>20 mm</td>
<td>0.096 mm</td>
</tr>
<tr>
<td>M3 (Silicon)</td>
<td>$148 \times 10^9$ Pa</td>
<td>0.28</td>
<td>20 mm</td>
<td>0.714 mm</td>
</tr>
</tbody>
</table>
5 Results

The four-step phase shift algorithm is used in this study with $\delta_1 = 0, 90, 180,$ and $270$ deg. A computer program was developed to control the translation of the beamsplitting grating as well as to capture the image taken by the charge-coupled device (CCD) camera. The four interferograms are acquired by the program in less than a second. Thus, the effects of most environmental disturbances are minimized. The U- and V-field interferograms are taken before the loading as initial fringes and also under loading. The corresponding phase-shifted fringe patterns for the U/V field under initial and loaded conditions are shown in Figs. 9 through 12, respectively. The phase maps for both initial and loaded conditions are then calculated according to Eq. (4) and shown in Figs. 9(e) and 10(e). The phase-shifted interferograms of the V-field before and under the loading, as well as the calculated phases, are shown in Figs. 11 and 12, respectively.

Close observation of Figs. 9 through 12 reveals the existence of intensity noise. As we discussed in Sec. 3, the noise may come from the imperfection of the specimen grating, such as defects in the reflection surface, etc. Image processing is necessary to obtain a smooth version of the phases. The actual phase change $\varphi_{\text{actual}}$ due to the mechanical loads is the subtraction of $\varphi_{\text{load}}$ and $\varphi_{\text{ini}}$. Figure 13 shows the reconstructed fringes from $\varphi_{\text{u, actual}}$ and $\varphi_{\text{v, actual}}$.

Clearly the U and V fields represented by the reconstructed fringe curves shown in Figs. 13(a) and 13(b) are very close to the center portion of the U and V displacement simulation results plotted in Figs. 8(b) and 8(c). Due to size of the laser beam, only the center portion of the specimen is imaged in Figs. 9 to 13. In other words, the phase shifting algorithm indeed provided reliable phase information of the mechanical loading without the requirement of the ideal initial null field. However, the details of $\varphi_u$ and $\varphi_v$ still suffer from the existing noise sources. CWT filtering is applied to both $\varphi_u$ and $\varphi_v$. The resulting ridges and phases from the CWT filtering are shown in Fig. 14. The improvement is obvious. For example, the vertical white lines cutting through the entire image to the left of the center of the image are eliminated. Also, the speckles in Figs. 13(a) and 13(b) disappear in Figs. 14(a) and 14(c).

The fringes shown in Figs. 9 through 14 are for the center of the specimen under loading. Since the displacement and the strain are symmetric due to the uniform beam and loading, the U-field ridge and phase of half of the specimen are shown in Figs. 15(a) and 15(b). The phase shown in Fig. 15(b) is further filtered by the DWT processing method, as discussed in Sec. 3.3. The contour of the resultant in-plane strain $\varepsilon_x$ is plotted in Fig. 15(c). Compared to the expected $\varepsilon_x$ obtained in the Ansys® simulation, as shown in Fig. 16, the effectiveness of the combination of phase shifting moiré and wavelet-based image processing.
algorithms is clear. From both the Ansys® simulation and moiré data, the minimum strain occurs at the top center of the specimen where the value is $7.7 \times 10^{-5}$ from moiré and $9 \times 10^{-5}$ from Ansys®. Similarly, the maximum strain occurs at the bottom center of the specimen where the value is $2.04 \times 10^{-4}$ from moiré and $2 \times 10^{-4}$ from Ansys®.

6 Conclusion

In this study, phase shifting moiré interferometry is implemented to obtain a rough phase map of the interferogram as well as reduce background noise to some extent by the subtraction of the shifted interferograms. The additional phase shifts are introduced by the translation of the beam-splitting grating. The program that simultaneously cooperates the grating position control and the data acquisition

![Fig. 12 V-field: under loading. (a) $I_1$ at $\delta_1=0^\circ$, (b) $I_2$ at $\delta_2=90^\circ$, (c) $I_3$ at $\delta_3=180^\circ$, (d) $I_4$ at $\delta_4=270^\circ$, and (e) $\Phi_{v,\text{load}}$.](image)

![Fig. 13 U-field $\varphi_{u,\text{actual}}$ and V-field $\varphi_{v,\text{actual}}$: (a) $\cos(\varphi_{u,\text{load}}-\varphi_{u,\text{ini}})$, and (b) $\cos(\varphi_{v,\text{load}}-\varphi_{v,\text{ini}})$.](image)

![Fig. 14 CWT results of U and V fields: ridge and filtered phase. (a) ridge U field, (b) $\varphi_u$, (c) ridge V field, and (d) $\varphi_v$.](image)

![Fig. 15 Calculated deformation and normal strain $\varepsilon_x$ of the right half of the specimen: (a) ridge U field, (b) $\varphi_u$, and (c) contour of $\varepsilon_x$.](image)
reduces the environmentally induced phase shift error. The phase representing the U and V deformation of the specimen under the uniform pressure loads is very close to the Ansys® simulated deformation plot. A continuous wavelet transform is used to remove the noise of the phase map obtained by phase shifting moiré interferometry. A significant improvement of the filtered phase map is demonstrated. The in-plane strain distribution is finally calculated by deriving the first derivative of the phase map. A discrete wavelet transform is used to smooth the resultant derivative map. The accuracy of the in-plane strain is demonstrated and supported by the comparison of the calculated strain contour and that from Ansys®.

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References


Heng Liu received the BS degree in electrical engineering from Tsinghua University, Beijing, China, in 1999, and MS and PhD degrees in electrical engineering from University at Buffalo, New York, in 2001 and 2003, respectively. She is currently doing functional MRI research with Medical College of Wisconsin.

Alexander N. Cartwright received the BS and PhD degrees in electrical and computer engineering from The University of Iowa in 1989 and 1995, respectively. He joined University at Buffalo in 1995, where he is now an associate professor in the Department of Electrical Engineering. His current research interests include spectroscopic characterization of semiconductor photonic devices, biophotonics, and electronic packaging.

Cemal Basaran received the MS degree in civil engineering from Massachusetts Institute of Technology, Cambridge, in 1988, and the PhD degree in engineering mechanics from University of Arizona, Tucson, in 1994. He joined University at Buffalo in 1995, where he is now an associate professor in the Department of Civil, Structural, and Environmental Engineering. His current research interests include electronic packaging, reliability of interconnects and interfaces, damage mechanics, finite element methods, and experimental mechanics.