# Analysis of Multilayered Microelectronic Packaging Under Thermal Gradient Loading

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***Abstract*—An accurate estimation of interfacial and axial stresses in multilayered structures is important in the design process of microelectronic packaging because these stresses drive the failure modes in the package. During manufacturing, microelectronic packaging devices usually suffer from severe thermal gradients. Design engineers often simplify the thermal gradient case as an isothermal loading case by averaging the temperature of the top and bottom of the microelectronic packaging device. Such simplification usually underestimates the stress level in the devices. With the analytical model presented in this paper, the stresses in multilayered microelectronic packaging devices subjected to thermal gradient loading can easily be predicted. It is shown that ignoring the thermal gradient in the package leads to underestimation of stresses.**

***Index Terms*—Die fracture, interfacial delamination, interfacial stress, laminated beams, microelectronic packaging, multilayered structure, thermal gradient.**

## I. INTRODUCTION

IN ELECTRONIC packaging, materials with different coefficients of thermal expansion (CTE) and mechanical properties are bonded together to form laminated structures, such as power electronics devices, circuit boards, and semiconductor devices. Thermal stresses that occur due to a CTE mismatch of the adhesively joined layers during manufacturing, machining, and field use can result in delamination failure or die fracture. When interfacial delamination is avoided by using strong adhesives usually die cracking becomes an issue to deal with. The rapid and reasonably accurate estimate of the stress level in these structures is very important for the design and reliability of laminated microelectronic packaging.

Many researchers studied failure mechanisms in multilayered microelectronic packaging. An extensive literature survey on the subject is given by Basaran and Zhao [1]–[3]. Timoshenko [24] is the first to study stresses in layered structures. He used elementary beam theory to obtain the curvature of a bimetallic beam due to a uniform temperature change. However, only the normal stresses in the thermostat strips were evaluated. It is the interfacial shearing and peeling stresses that are responsible for the structural integrity of the thermostat. Hayashi [10] presented the first analytical model that focused on the computation of interfacial shear stress. Hayashi's model was based on the implicit assumption that the in-plane stresses within a given layer are independent of the thickness coordinate. Grimado [9] neglected the shear deformations in the beam elements in his study. All of the materials were assumed to transmit only longitudinal shear and out-of-plane normal stress, the adhesive layers were assumed to be thin, and the bonded structure was considered to be free of initial stresses. Chen and Nelson [8] derived the closed-form solution of stress distribution in bonded materials due to their thermal expansion mismatch for assemblies used in electronic devices. A number of simplifying assumptions were made to provide good approximations. Their paper presented two- and three-layered analytical modes with and without bending to show the relative importance of layer thickness and material parameters. Suhir [16]–[23] improved on Timoshenko's bithermostat beam theory with relatively simple calculations using longitudinal and transverse interfacial compliances, widely known as Suhir solution, which is the most commonly used benchmark analytical procedure in the electronic packaging literature. Pao and Eisele [13] extended Suhir's bimetal thermostat model to multilayered thin stacks without imposing any additional assumptions on the interface.

Chen *et al.* [7] took a significantly different approach, which satisfied the boundary conditions at the free edges of a laminated beam. The analysis was based on two-dimensional elasticity theory and the variational theorem of complementary energy presented by Washizu [26] under the assumption of linear distribution of longitudinal normal strain through the thickness of each layer. A similar approach was applied by Williams [30] and showed good agreement with the results of Chen *et al.* [7]. Bogy [5], [6], Hein and Erdogan [11] and Yin [31] discussed stress singularities at the bimaterial interfaces near the free edges. Such stress singularities cannot be directly determined by the standard elastic finite element analysis alone. Asymptotic analysis is needed around the junction point to determine the stresses in the near-tip stress field [12], [14], [15]. Shih and Asaro [14], [15] studied elastic-plastic analysis of cracks on bimaterial interfaces, where they showed mesh dependence of finite element analysis and necessity of asymptotic analysis. In real life, such stress singularity $(1/r)$ cannot exist physically. Once the stress level reaches the yield strength, ductile materials yield and brittle materials crack at their ultimate tensile strength as a result the stresses are re-distributed to neighboring points. Basaran and Zhao [1] have shown that when damage mechanics based elastic-viscoplastic

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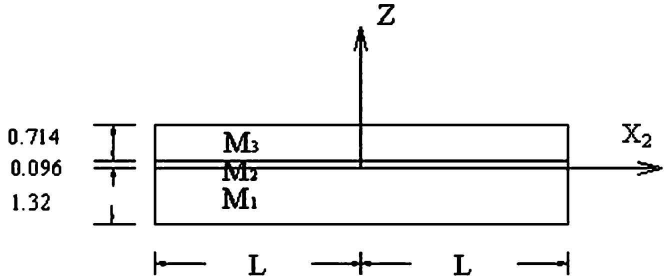

Fig. 1. Idealized model geometry.

material models are used in the finite element method stress singularity ceases to be a significant issue.

Unfortunately, most of the analytical models proposed in the literature have not been verified experimentally for actual microelectronic packaging structures with very thin layer stacked on top of a thick substrate where size makes the strain measurement process very difficult. Moreover, they also assume a uniform isothermal loading over the entire structure.

Wen and Basaran [27]–[29], (2005) developed an analytical model for multilayered microelectronic structures based on the refined plate theory without the assumption of perfectly bounded interfaces. This model considers each layer as a beam-type plate with orthotropic material properties. It has been verified in the lab by using a high sensitivity Moiré interferometry. Testing was conducted on an actual microelectronic package. This model can also be applied for thermal gradient. In this paper the difference between uniform isothermal average loading and thermal gradient loading is investigated. The details of the formulation are presented in Appendix A. In the following section, a case study on a multilayered package is presented.

## II. CASE STUDY OF A MULTILAYERED MICROELECTRONIC PACKAGE

In order to investigate the influence of simplifying the temperature field from a gradient to isothermal average, a three-layered microelectronic structure is considered. Fig. 1 shows the idealized model geometry used in this study. The first layer is BT, the second layer is eutectic Pb/Sn solder, and the third layer is silicon. The proposed analytical model has been verified by Wen and Basaran (2003), [29] using high sensitivity moiré interferometry. The thermal gradient is $\Delta T = 50Z{+}100$. The temperature of the structure is 140.5 °C at the top and 34 °C at the bottom. The average temperature of the top and bottom is 87 °C. Material properties and dimensions are shown in Table I.

The following boundary conditions were used for this problem:

At $x_2 = 0$

$$U_2^i = 0, \quad \Phi_2^i = 0, \quad Q_2^i = 0 \quad \text{where } i = 1 \textit{ to } 3.$$

At $x_2 = L$

$$N_2^i = 0, \quad M_2^i = 0, \quad Q_2^i = 0 \quad \text{where } i = 1 \textit{ to } 3.$$

TABLE I
MATERIAL PROPERTY AND DIMENSIONS

| | $M_1$(BT) | $M_2$(Pb/Sn solder) | $M_3$(silicon) |
|---|---|---|---|
| $E_1$(Gpa) | 17.5 | 24.3 | 112 |
| $E_2$(Gpa) | 10.4 | 24.3 | 148 |
| $E_3$(Gpa) | 4.7 | 24.3 | 168 |
| $G_{12}$(Gpa) | 3.54 | 9.2 | 46.2 |
| $G_{13}$(Gpa) | 9.04 | 9.2 | 33.2 |
| $G_{23}$(Gpa) | 1.58 | 9.2 | 51.7 |
| $v_{12}$ | 0.32 | 0.32 | 0.28 |
| $v_{13}$ | 0.32 | 0.32 | 0.28 |
| $v_{23}$ | 0.32 | 0.32 | 0.28 |
| $\alpha_1$(e-6/°C) | 16 | 24.7 | 3 |
| $\alpha_2$(e-6/°C) | 16 | 24.7 | 3 |
| $\alpha_3$(e-6/°C) | 16 | 24.7 | 3 |
| h (mm) | 1.32 | 0.096 | 0.714 |
| 2L(mm) | | 20.5 | |

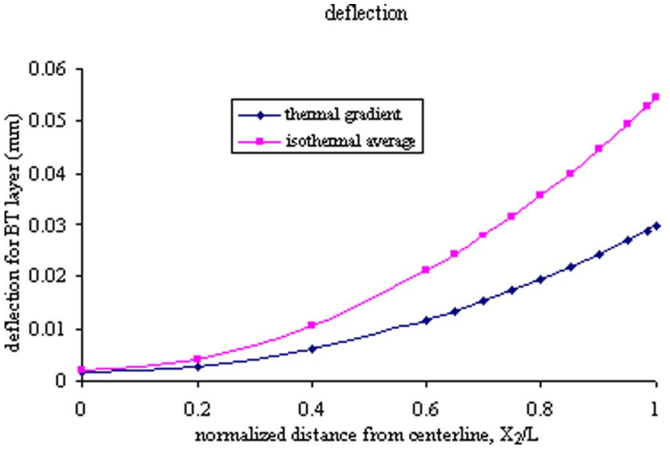

Fig. 2. Comparison of the deflection for BGA module under thermal gradient and isothermal average loadings.

## III. RESULTS AND DISCUSSION

Fig. 2 shows the deflection of the package for thermal gradient loading and isothermal average loading. The deflection for isothermal average is more than that for thermal gradient by 82%. The simplification of thermal gradient to isothermal average significantly overestimates the deflection. Second derivative of the deflection is directly proportional to the bending moment in the beam. Hence the flexural stresses and curvature are grossly underestimated.

Figs. 3 and 4 show the distribution of axial normal stresses along the longitudinal direction for the BT and silicon layers under thermal gradient and isothermal average loadings, respectively. It is obvious that the axial normal stress in the BT layer is overestimated by 16% and the axial normal stress in the silicon layer is underestimated by 8% due to simplified temperature loading.

Figs. 5 and 6 show the shear stress distribution along interfaces one and two for thermal gradient and isothermal average loadings. The simplification from thermal gradient to isothermal average leads to underestimation of the shear stress by 53% for interface 1 and 23% for interface 2 near the free edge. Figs. 7 and 8 show the distribution of peeling stress at interfaces 1 and 2. The maximum peeling stress is underestimated by 44% for interface 1 due to this simplification. On the other hand, the peeling stress is overestimated by 85% for interface 2 near the free edge.

## IV. CONCLUSION

In this paper, it is shown that simplifying the thermal loading to isothermal temperature field can significantly underestimate the peeling and shear stresses in a laminated structure.

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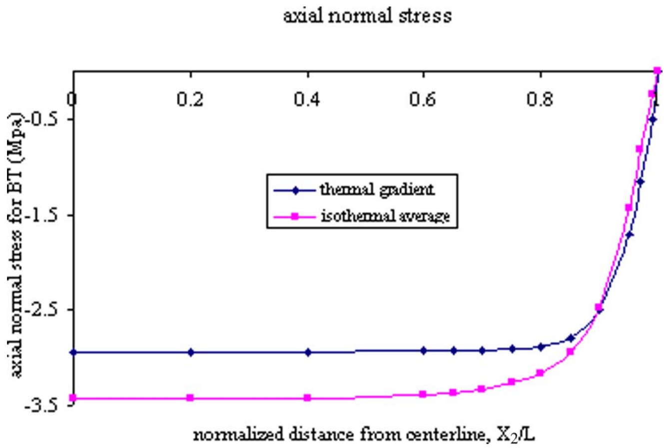

Fig. 3. Comparison of the axial normal stress along the longitudinal direction for BT layer for BGA module under thermal gradient and isothermal average loadings.

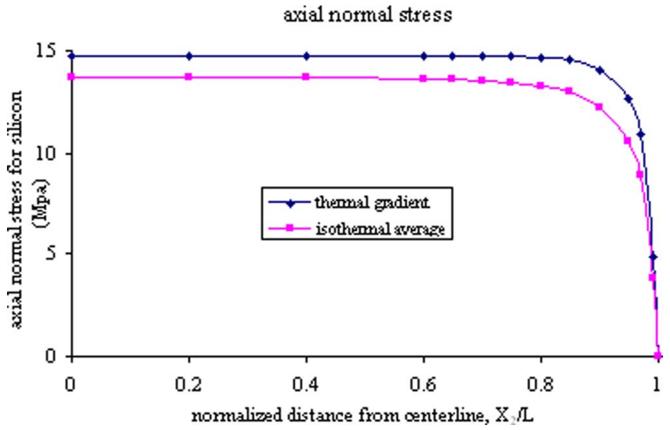

Fig. 4. Comparison of the axial normal stress along the longitudinal direction for silicon layer for BGA module under thermal gradient and isothermal average loadings.

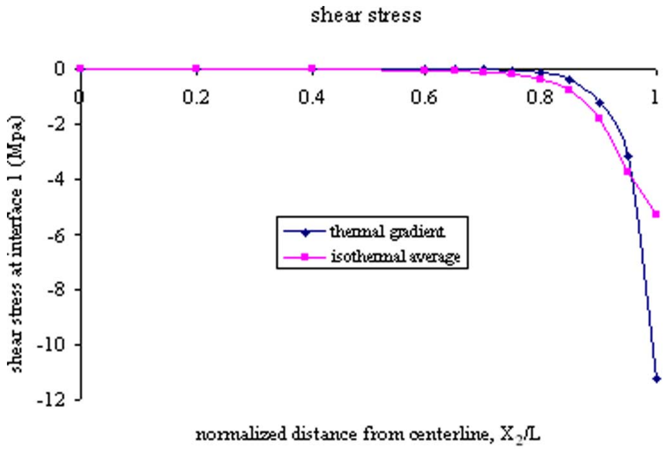

Fig. 5. Comparison of the shear stress at interface 1 for BGA module under thermal gradient and isothermal average loadings.

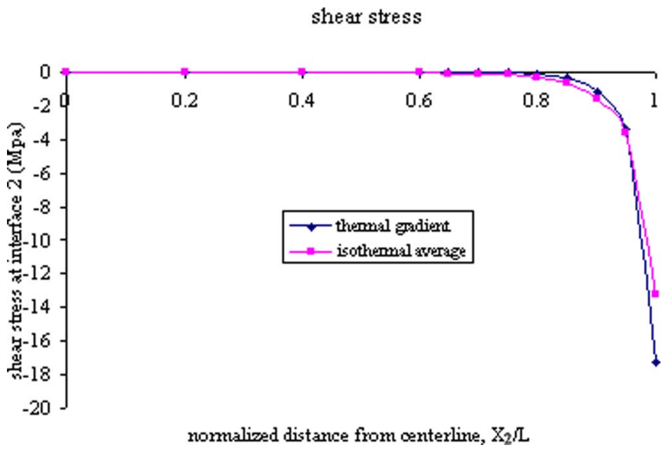

Fig. 6. Comparison of the shear stress at interface 2 for BGA module under thermal gradient and isothermal average loadings.

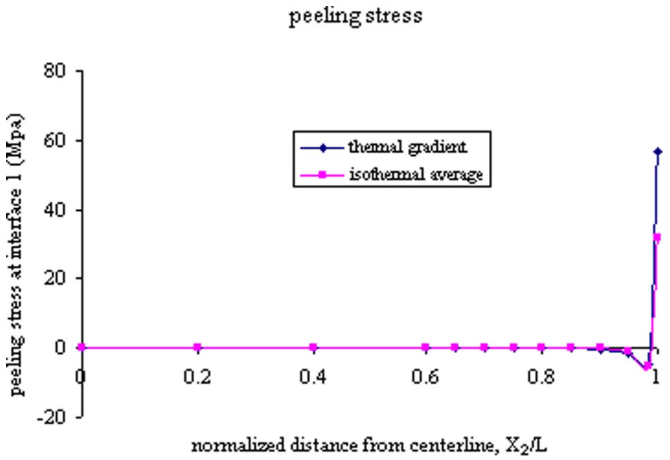

Fig. 7. Comparison of the peeling stress at interface 1 for BGA module under thermal gradient and isothermal average loadings.

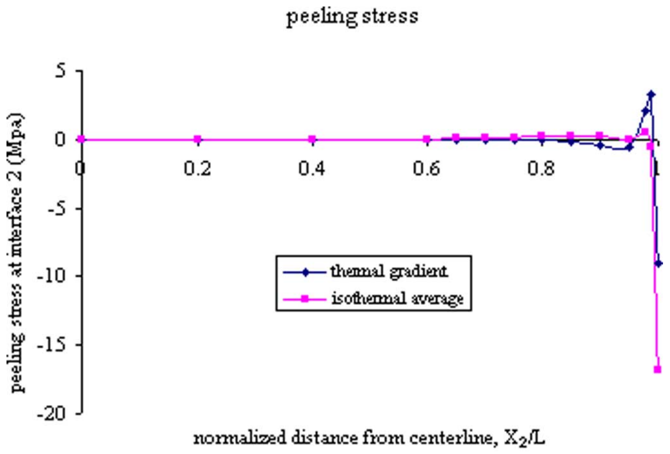

Fig. 8. Comparison of the peeling stress at interface 2 for BGA module under thermal gradient and isothermal average loadings.

Compared to other analytical models that can only be applied to the isothermal loading cases, the model presented in this paper can also be applied to the thermal gradient cases. During manufacturing and in service microelectronic packaging devices usually experience severe thermal gradients. The stress levels in microelectronic packaging devices under thermal gradient loading can easily and accurately be predicted with this analytical model.

The underestimation of the peeling stress in the design stage usually leads to delamination failure at the interfaces. On the

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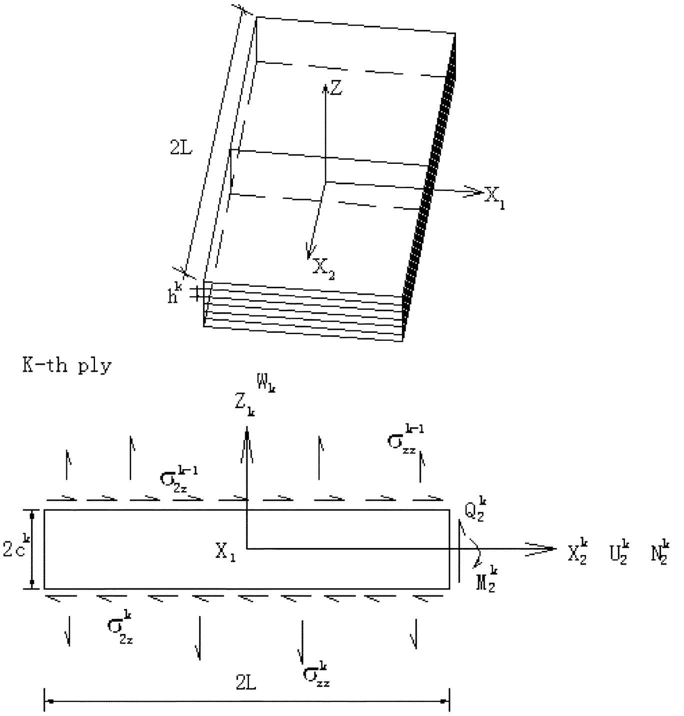

Fig. 9. Generic laminated beam-type plate.

other hand, underestimation of axial normal stress is usually responsible for the die fracture failure in the silicon layer.

## APPENDIX A
## SUMMARY OF THE FORMULATION OF THE ANALYTICAL MODEL

Consider a $N$-layered laminated plate as shown in Fig. 9. We assume that due to boundary constraints the plate bends in one direction at a time. We will call this a beam-type plate, which is true for most microelectronic packaging structures. The equilibrium and the constitutive equations for the beam-type plate theory yield a set of 8N equations in terms of 8N variables (2N displacements, N rotations, 3N force resultants, and 2N moment resultants). This set is supplemented by an additional set of $2(\mathrm{N}-1)$ equations, which are necessary for the simultaneous solution of $2(\mathrm{N}-1)$ interfacial stresses. The displacement is assumed to be continuous at $(\mathrm{N}-1)$ interfaces. Among this set of equations the force-resultant and moment-resultant variables can be eliminated with the aid of constitutive equations. As a result we have a set of $(5\mathrm{N}-2)$ coupled differential equations to be solved for 2N displacement variables, N rotations, and $2(\mathrm{N}-1)$ interfacial stresses.

Assuming there are no body forces and couple stresses acting on the system, for a beam-type plate, force and moment equilibrium equations for a kth layer in terms of stresses can be given by

$$N_{2,2}^k + n_2^k = 0 \tag{1}$$

$$M_{2,2}^k + c_k m_2^k - Q_2^k = 0 \tag{2}$$

$$Q_{2,2}^k + q^k = 0 \tag{3}$$

where N indicates axial force, M indicates moment and Q indicates shear force per unit width. The comma identifies differentiation with respect to the axis number after the comma. Superscript k identifies layer number. The difference in interfacial stresses between layers k and $(\mathrm{k}-1)$ yield the stress imposed on each layer, which can be given by

$$n_2^k = \sigma_{2z}^{k-1}(x_2, c_k) - \sigma_{2z}^k(x_2, -c_k) \tag{4a}$$

$$m_2^k = \sigma_{2z}^{k-1}(x_2, c_k) + \sigma_{2z}^k(x_2, -c_k) \tag{4b}$$

$$q^k = \sigma_{zz}^{k-1}(x_2, c_k) - \sigma_{zz}^k(x_2, -c_k). \tag{4c}$$

For a beam-type plate all derivatives with respect to $x_1$ are zero. The thickness of the kth ply is $2\mathrm{c}^\mathrm{k}$. Superscript k, which identifies the generic ply, will be dropped in the subsequent part for convenience.

Laminated microelectronic structures usually have orthotropic material properties. And the loading is usually thermal gradient. Starting with Valisetty's [25] definition of a beam-type plate, we introduce thermal gradient, orthotropic material properties and interfacial compliances. Introducing interfacial compliances will allow us to have interfaces with a certain stiffness value compared to perfectly bonded rigid interfaces. In the microelectronics industry there is a strong desire to design packages with flexible interfaces to decrease interfacial stresses. Influence of interfacial compliances on stress distribution in interfaces and in laminated layers is presented in Basaran and Wen [4]. Introducing orthotropic material properties to beam-type plate equations yields the following constitutive relations;

$$\frac{N_i}{h} = -\overline{C}_{ij}\Delta T \alpha_j + \overline{C}_{i2} U_{2,2} + K_{ni} c n_{2,2} + K_{pi} p; \quad i, j = 1, 2 \tag{5a}$$

$$\frac{M_i}{\overline{I}} = -\overline{C}_{i2} W_{,22} + K_{mi} m_{2,2} + K_{qi} q/c \tag{5b}$$

$$\Phi_2 + W_{,2} = \frac{c^2}{2\overline{I}} S_{44}\left(Q_2 - \frac{1}{3} c m_2\right) \tag{5c}$$

where

$$\begin{aligned}
K_{mi} &= (3\overline{C}_{i2} S_{3j} \overline{C}_{j2}/\overline{C}_{22} - 2\overline{C}_{i2} S_{44} + 2\overline{C}_i)/20; \\
&\quad i, j = 1 \textit{ and } 2 \\
K_{qi} &= (3\overline{C}_{i2} S_{3j} \overline{C}_{j2}/\overline{C}_{22} - 12\overline{C}_{i2} S_{44} + 12\overline{C}_i)/20 \\
K_{ni} &= (\overline{C}_{i2} S_{3j} \overline{C}_{j2}/\overline{C}_{22} + \overline{C}_{i2} S_{44} + 2\overline{C}_i)/12 \\
K_{pi} &= \overline{C}_i/2 \\
p &= \sigma_{zz}(x_2, c) + \sigma_{zz}(x_2, -c) \\
h &= 2c, \quad \overline{I} = 2c^3/3 \\
\overline{C}_{11} &= C_{11} - \frac{C_{13}C_{31}}{C_{33}}, \quad \overline{C}_{12} = C_{12} - \frac{C_{13}C_{32}}{C_{33}}, \\
\overline{C}_{21} &= C_{21} - \frac{C_{23}C_{31}}{C_{33}}, \quad \overline{C}_{22} = C_{22} - \frac{C_{23}C_{32}}{C_{33}} \\
\overline{C}_1 &= \frac{C_{13}}{C_{33}}, \quad \overline{C}_2 = \frac{C_{23}}{C_{33}} \\
C_{ij} &: \text{stiffness coefficients of orthotropic materials}
\end{aligned} \tag{6}$$

| | |
|---|---|
| $\alpha_j$ | coefficient of thermal expansion of $k$th layer in direction $j$; |
| $\Delta T$ | thermal gradient; |

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| | |
|---|---|
| U, W | displacement components in the $x_2$ and $z$ directions, respectively, at $z = 0$ surface; |
| $\Phi_2$ | the rotation of a normal to the reference surface ($z$ =0). |

Solution of the differential equations given by the equilibrium equations and interfacial compatibility conditions for the classical plate theory with beam-like behavior assumptions yield the following stress and displacement distribution equations;

$$\begin{aligned}\sigma_i = &\frac{1}{h}N_i + \frac{1}{2h}K_i n_{2,2}(z^2 - c^2/3) + \frac{z}{\bar{I}}M_i \\ &+ \frac{1}{6\bar{I}}K_i(cm_{2,2}+q)(z^3 - 3c^2 z/5); \quad i,j=1,2 \end{aligned} \tag{7a}$$

$$\sigma_{2z} = \frac{z}{h}n_2 + \frac{c}{6\bar{I}}m_2(3z^2 - c^2) - \frac{1}{2\bar{I}}Q_2(z^2 - c^2) \tag{7b}$$

$$\sigma_{zz} = \frac{1}{2}p - \frac{1}{2h}n_{2,2}(z^2 - c^2) + \frac{z}{h}q - \frac{1}{6\bar{I}}(cm_{2,2}+q)(z^3 - c^2 z) \tag{7c}$$

where

$$K_i = \overline{C}_{i2}S_{3j}\overline{C}_{2j}/\overline{C}_{22} + \overline{C}_{i2}S_{44} - \overline{C}_i; \quad i,j = 1,2 \tag{8}$$

| | |
|---|---|
| $\sigma_1$ | normal stress in the $x_1$ direction in any layer; |
| $\sigma_2$ | normal stress in the $x_2$ direction, in any layer; |
| $\sigma_{2z}$ | shear stress in the $x_2 - z$ plane; |
| $\sigma_{zz}$ | transverse normal stress in the thickness coordinates $z$ direction, peeling stress; |

$$\begin{aligned} w = &W + S_{3j}\left(N_j\frac{z}{h} + M_j\frac{z^2}{2\bar{I}}\right) + S_{3j}K_j\frac{1}{6h}n_{2,2}(z^3 - c^2 z) \\ &+ S_{3j}K_j\frac{cm_{2,2}+q}{6\bar{I}}(z^4/4 - 3c^2z^2/10) \\ &+ S_{33}\left(\frac{z}{2}p + \frac{z^2}{2h}q\right) - S_{33}\frac{1}{6h}n_{2,2}(z^3 - 3c^2 z) \\ &- S_{33}\frac{cm_{2,2}+q}{6\bar{I}}(z^4/4 - c^2z^2/2) + z\Delta T\alpha_z \quad j = 1,2 \end{aligned} \tag{9a}$$

$$\begin{aligned} u_2 = &U_2 - zW_{,2} - S_{3j}\left(N_j\frac{z^2}{2h} + M_j\frac{z^3}{6\bar{I}}\right)_{,2} \\ &- S_{3j}K_j\left\{\frac{n_{2,22}}{6h}(z^4/4 - c^2z^2/2)\right. \\ &\qquad \left. + \frac{cm_{2,22}+q_{,2}}{6\bar{I}}(z^5/20 - c^2z^3/10)\right\} \\ &- S_{33}\left\{\frac{z^2}{4}p_{,2} - \frac{1}{6h}n_{2,22}(z^4/4 - 3c^2z^2/2) + \frac{z^3}{6h}q_{,2}\right. \\ &\qquad \left. - \frac{cm_{2,22}+q_{,2}}{6\bar{I}}(z^5/20 - c^2z^3/6)\right\} \\ &+ S_{44}\left\{\frac{z^2}{2h}n_2 + \frac{1}{6\bar{I}}cm_2(z^3 - c^2 z)\right. \\ &\qquad \left. - \frac{1}{6\bar{I}}Q_2(z^3 - 3c^2 z)\right\} \end{aligned} \tag{9b}$$

where

| | |
|---|---|
| w, $u_2$ | the displacement components in the $z$ and $x_2$ coordinate directions, respectively. |

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